Computing Satellite Passes on a Spread-sheet Program

J.Köppen,
Observatoire de Strasbourg
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What we want to simulate

To interpret data obtained during the pass of a satellite over our ground station, we would like to know the positions, azimuth and elevation, and the distance to the satellite at all times during the pass. For instance, the distance allows computation of the expected signal strength of the satellite's transmitter, so that we could compare the expected values as a function of time, along with our measurements.

The base for the equations is the geometry of the circular orbit of the satellite around the spherical Earth. The satellite moves with uniform speed on its orbit, while the Earth beneath rotates around its axis. Spherical trigonometry is used to work out all the angles we are interested in.

The input parameters

We need these input parameters:

- Geographic position of the observer **olong**, **olat**, and the height **ho** above sea level. For our equations we shall use the convention that Eastern longitudes and Northern latitudes are taken positive. It is useful to compute from them the position in Cartesian coordinates
  $$xo = (R_{EARTH} + ho) * \sin(olong) * \cos(olat)$$
  $$yo = (R_{EARTH} + ho) * \cos(olong) * \cos(olat)$$
  $$zo = (R_{EARTH} + ho) * \sin(olat)$$
  with the radius of the Earth R_{EARTH} = 6371 km. For Strasbourg we may safely neglect ho = 0.2 km.
- The satellite orbit (assuming a circular orbit):
  - Altitude: **h** (given in km)
  - Inclination: **i**
  - Period: **T** (given in minutes; in principle this could also be computed from the orbital altitude…)
  - The **longitude of the last ascending crossing of the equator**: ☊ (for convenience, this quantity must be specified for the orbit under consideration; it is obtained from the orbital prediction program – for example http://astro.u-strasbg.fr/~koppen/PassFinder/ – but also see last Section how one can derive this datum from the pass information furnished by the NOVA software which steers the antenna rotators)

The equations

For any time t after the equator crossing, we first compute the angle in the satellite's orbital plane between its current position and the crossing of the equator:

$$\alpha = 360° * t/T \qquad \text{(or } 2\pi * t/T \text{, if we prefer to measure angles in radians)}$$

Then we compute two help quantities

$$hx = \cos(i) * \sin(\alpha)$$
$$hy = \cos(\alpha)$$

and the angle

$$\Delta l = \text{atan2}(hy, hx)$$

Thus we can compute the geographical latitude and latitude of the satellite at that moment:

$$lat = \text{atan}(\tan(i) * \sin(\Delta l))$$

$$long = \Omega + \Delta l - t*360°/(24*60) \qquad \text{as we measure times in minutes}$$

The last term describes how much the Earth has rotated during the time passed since the satellite crossed the equator. Next we compute the satellite's position in Cartesian space:

$$x = (R_{EARTH} + h) * \sin(long) * \cos(lat)$$
$$y = (R_{EARTH} + h) * \cos(long) * \cos(lat)$$
$$z = (R_{EARTH} + h) * \sin(lat)$$

and its distance (or range) to the observing station from

$$d^2 = (x-xo)^2 + (y-yo)^2 + (z-zo)^2$$

The elevation angle above the observer's horizon can then be worked out

$$EL = \text{asin}((R_{EARTH}+h)^2 - R_{EARTH}^2 - d^2) / (2 R_{EARTH} d))$$

The azimuth is obtained with

$$hx = \sin(long-olong)$$
$$hy = \cos(olat)*\tan(lat) - \sin(olat)*\cos(long-olong)$$

$$AZ = \text{atan2}(hy, hx)$$

Finally, we can also compute the frequency shift from the change of the distance during one time step Δt, if we take the current distance d(t) and the previous one $d(t-\Delta t)$

$$\Delta f = - f_0 * (d(t) - d(t-\Delta t))/ \Delta t /(60*300)$$

If f_0 is the transmitter's frequency in MHz, d in km, and Δt in minutes, the result will be in convenient units of kHz.

Estimate of the Signal Strength

If the details of the communication link (transmitter power, antenna gains, …) are known, one may compute the signal strength in absolute power at the receiver.

signal_level_dBm = EIRP_dBm – 20 * log10(0.3/(4π f_MHz d_km)) + RXgain_dB

Here, RXgain_dB is the sum of all gains ahead of the receiver's antenna socket: the receiving antenna gain and the preamplifier, minus all losses from cables etc.

But we can also keep this a bit more general and independent of these details by simply computing the dependence of the signal on the distance, and scale the result with some convenient factor:

signal_level = signal_offset – 20 * log10(d /1000 km)

The result is in dB, and we may choose the signal_offset to match the measured level. Also, it is more convenient to compute the signal level only for positive elevation angles!

How to solve them

We simply evaluate the formulae to compute for all times after the crossing of the equator (during, say, one orbital period) all the positions of the satellite, and then plot the results. We use a time step Δt as small as needed to obtain sufficiently smooth curves in the plots.

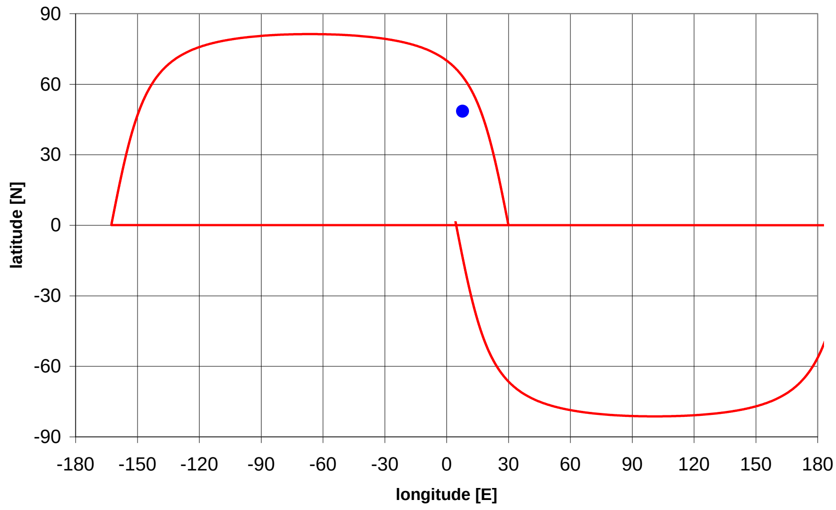
How to test the program

There are no fundamental formulae with which we could rigorously test the program, except compare with data of some other calculation which had been proven to be correct. We can only verify that each step of the procedures gives the meaningful and correct results. For this we use the geometry of the problem to identify certain results the program must reproduce:

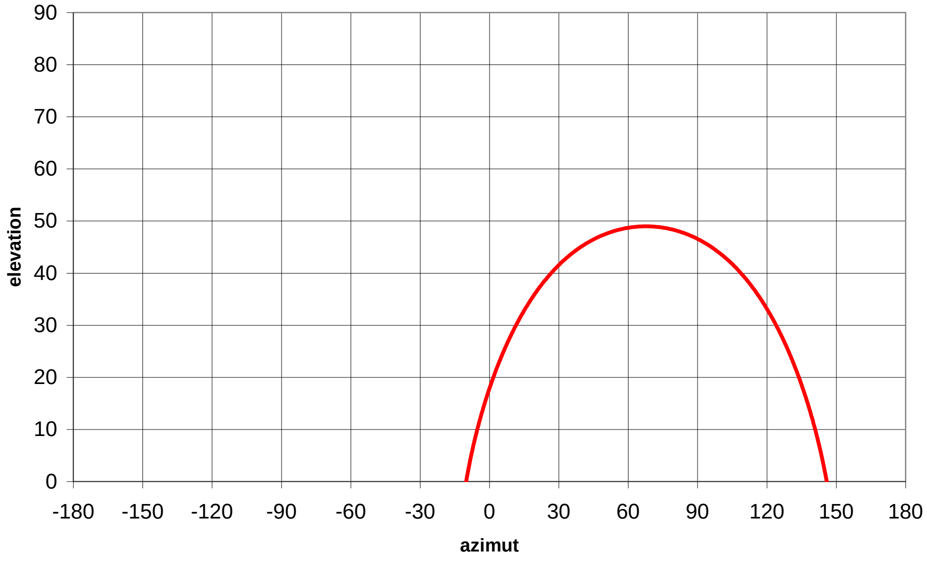
- At time zero, the longitude should be equal to that of the equator crossing, and the latitude should be zero. Also, for the time immediately after, latitude and longitude should behave consistent with the orbital orientation
- … and similar considerations, such as if the satellite passes east of the station, then the azimuth should be less than 180°, etc. …
- in most programs (including Excel) the trigonometric functions (sin, cos, tan) need as arguments the angle in radians: angle → angle*π/180°. Likewise, the inverse functions have radians as result.
    - One can either always include this conversion in the argument or simply work with radian angles only, and convert them into degrees only at the output…
- The function atan2(x,y) is very useful, because its output is an angle that has a range of the full circle; however, the order of the arguments might be different in different programs and computer languages. It is advisable to check it. In the formulae above, I used the version I found necessary for Excel.

Here is a test calculation with the input parameters:

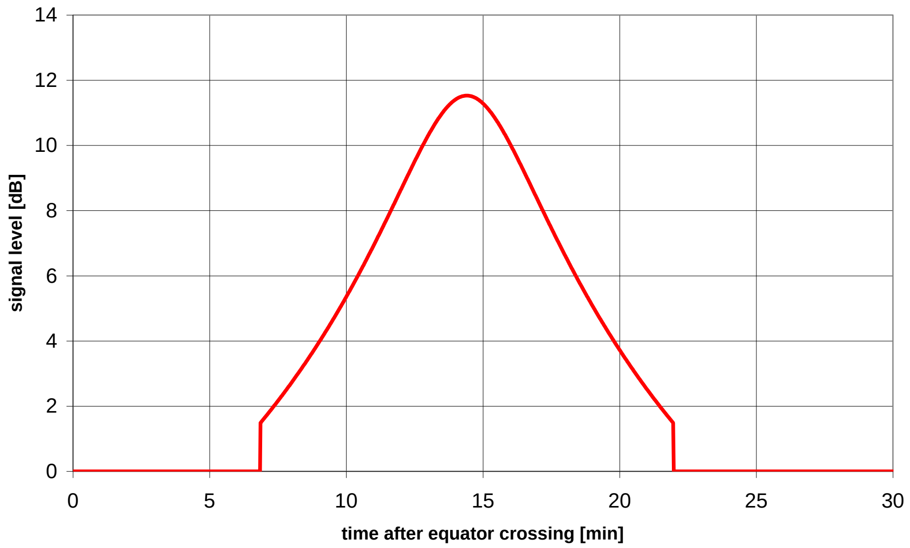
Observer	
longitude E	7.736778
latitude	48.523105
Satellite	
altitude [km]	830
inclination	98.7
period [min]	101.4
frequency [MHz]	436
asc.node	30
Constants	
Earth radius [km]	6371
time step [min]	0.02



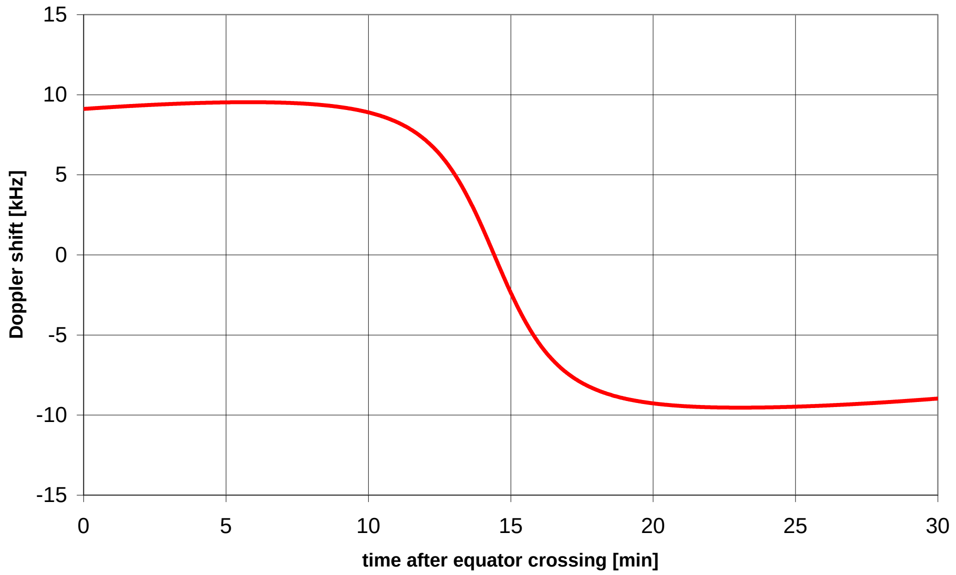
The satellite starts at 30°E and moves in north-westerly direction. When it crosses the equator going southward, the curve makes a jump in longitude from -160° of about 200°E. When it passes over Strasbourg, it will be in the Eastern sky, but it will not pass directly overhead:



If one assumes an offset of 12 dB for the signal, one gets this time variation of the signal:

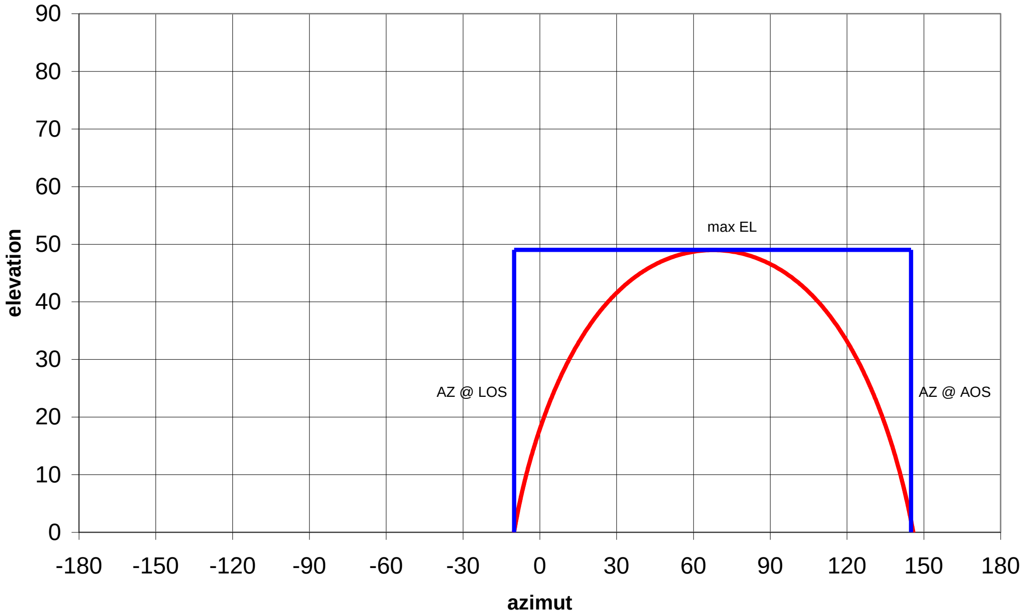


and the Doppler shift

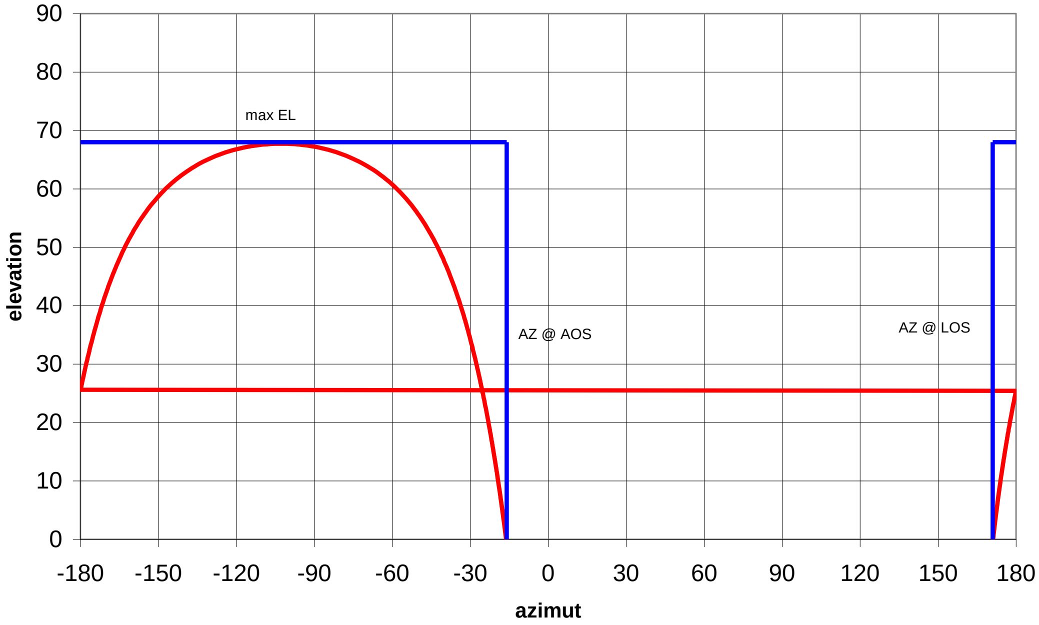


How to get the Equator Crossing Longitude

The NOVA software does not furnish directly the equator crossings, but one can move the mouse on the map to that position and read off the longitude. However, it is also quite easy to derive this parameter from the predicted azimuths at AOS and LOS and the maximum elevation of that pass. We draw a box in the azimuth-elevation plot and play with the longitude of the equator crossing until the predicted curve of the pass fits into the box, as shown below for AZ @ AOS = 145°E, max EL = 49°, and AZ @ LOS = 10° W = -10°:



The only thing to watch out is the case when the satellite would pass over the southern direction (AZ = 180° = -180°). Here, we have to draw the box in a different way (AZ @ AOS = -16°, max EL = 69°; AZ @ LOS = 171°):



Here, we shan't bother about the cosmetic imperfection that the predicted curve jumps across the plot from AZ = –180° to +180°!