A&A manuscript no.
(will be inserted by hand later)

Your thesaurus codes are:
...

ASTRONOMY
AND
ASTROPHYSICS
10.9.1996

Analysis of the Condensation and evaporation of interstellar clouds in 'chemodynamical' models of galaxies

J. Köppen[1,2,3], **C. Theis**[1], **and G. Hensler**[1]

[1] Institut für Astronomie und Astrophysik der Universität, D–24098 Kiel, Germany
[2] Equipe 'Evolution Galactique', URA 1280, Observatoire de Strasbourg, 11 rue de l'Université, F–67 000 Strasbourg, France
[3] International Space University, Parc d'Innovation, F-67400 Illkirch, France

September 10, 1996

Abstract. no abstract

Key words: —

1. Introduction

This contains the detailed description of the analysis of the models, which is not contained in the proper paper. Note: I did not remove the small misprints etc. so for the first sections, consult the paper itself.

2. The model for the ISM

2.1. The description used in CDE models

In the present CDE models of galaxies, one considers four components: the hot intercloud gas (named hereafter 'gas', with a mass density g), the gas in the form of clouds ('clouds' c), as well as massive stars (s), and low mass stars and remnants (r). These interactions between the components are taken into account: Star formation, gas return from dying stars, evaporation of clouds, condensation of gas onto clouds, (radiative or mechanical) heating of the gas by massive stars, radiative cooling of the gas. The time evolution of the mass densities of the components is described by the following equations:

$$\dot{g} = s\eta/\tau + Ec - Kg \quad (1)$$

$$\dot{c} = -\Psi - Ec + Kg \quad (2)$$

$$\dot{s} = \xi\Psi - s/\tau \quad (3)$$

$$\dot{r} = (1-\xi)\Psi + (1-\eta)s/\tau \quad (4)$$

Throughout the paper, we shall use the units parsec, 10^6 years, and solar masses.

Send offprint requests to: J. Köppen

Star formation is described by the stellar birth function $\Psi(c, T_c)$ used in the form of Köppen et al. (1995)

$$\Psi(c, T_c) = C_n\, c^n\, \exp(-T_c/1000\,\mathrm{K}) \quad (5)$$

Normally we use a quadratic dependence on density ($n = 2$). The exponential factor involving the temperature T_c of the cloud gas describes what fraction of a cloud is in the form of star forming molecular clumps.

The mass returned to the interstellar gas by dying massive stars (with a life-time $\tau = 10$ Myr) is taken to be the fraction $\eta = 0.9$ of the stellar mass.

Of all stars born, the fraction $\xi = 0.1$ is in the form of massive stars.

The remaining terms pertain to evaporation of clouds, whose rate coefficient E can be a function of densities and temperatures, and condensation of gas into clouds (coefficient K).

In the formulations of Hensler & Burkert (1991) and Theis et al. (1992) the cloudy medium is composed of clouds which have identical properties (radius R_c, mass M_c, density ρ_c) and which are embedded in the (hot) intercloud gas. Pressure equilibrium is assumed, which gives for the volume filling factor of the cloudy medium:

$$f_c = \frac{cT_c/\mu_c}{gT_g/\mu_g + cT_c/\mu_c} \quad (6)$$

with the mean molecular masses μ_c, μ_g in the two gas phases. For simplicity, we set $\mu_c = \mu_g = \mu_p$ equal to one proton mass, in all what follows. Then the true mean density ρ_c within a single cloud is given by

$$\rho_c = c/f_c \quad (7)$$

Assuming that the clouds are just unstable to the Jeans criterion, the mass is determined by

$$M_c = \frac{4\pi\rho_c}{3}R_c^3 = \sqrt{\frac{3}{4\pi\rho_c}}\left(\frac{k_\mathrm{B}T_c}{0.2G\mu_c}\right)^{1.5} \quad (8)$$

with Boltzmann's constant $k_{\rm B}$ and gravitational constant G. Hence the radius of the cloud is

$$R_c = \sqrt{\frac{3}{4\pi}\frac{k_{\rm B}T_c}{0.2G\mu_c}} \quad (9)$$

$$R_c[\mathrm{pc}] = 2.98 \cdot T_c^{1/2} \cdot (\rho_c[\mathrm{M}_\odot/\mathrm{pc}^3])^{-1/2} \quad (10)$$

Depending on its properties, such a cloud evaporates into the surrounding medium due to thermal conduction from the ambient hot gas or the gas condenses onto it. McKee & Cowie (1977) give a criterion for this behaviour: If the quantity defined as

$$\sigma_0 = \left(\frac{T_g}{1.54\,10^7\mathrm{K}}\right)^2 (R_c[\mathrm{pc}])^{-1} \left(n_g[\mathrm{cm}^{-3}]\right)^{-1} \quad (11)$$

$$= 1.246\,10^{-16} \cdot T_g^2 \cdot (R_c[\mathrm{pc}])^{-1} \cdot (g[\mathrm{M}_\odot/\mathrm{pc}^3])^{-1} \quad (12)$$

is smaller than 0.03, condensation occurs, otherwise the clouds evaporate. Since the clouds are assumed to have identical properties, the whole system switches between condensation and evaporation, depending on the criterion. Samland et al. (1996) consider a cloud population with a spectrum of masses, and therefore have at any time clouds that evaporate as well as those condensing. We discuss the implications in Sect. **??**

The mass loss rate per cloud due to evaporation is:

$$|\mathcal{M}_c|[\mathrm{g/s}] = 2.75\,10^4\,T_g^{2.5} \cdot (R_c[\mathrm{pc}])^{-1} \quad (13)$$

$$|\mathcal{M}_c|[\mathrm{M}_\odot/\mathrm{Myr}] = 2.75 \cdot 1.5795\,10^{-14} \cdot T_g^{2.5} \cdot (R_c[\mathrm{pc}])^{-1} \quad (14)$$

For $\sigma_0 > 1$ the mass loss rate decreases with increasing σ_0. In what follows, we do not consider this additional detail, as it alters the behaviour of the models only slightly. Likewise, we do not consider the modifications introduced by McKee & Begelman (1990). From the number density of the clouds

$$n_c = c/M_c \quad (15)$$

one gets the rates for evaporation and condensation

$$Ec = |\mathcal{M}_c| n_c \quad (16)$$

$$Kg = |\mathcal{M}_c| n_c (0.03/\sigma_0) \quad (17)$$

We note that with $\sigma_0 = 0.03$, on the border between evaporation and condensation behaviour, the rates are equal:

$$Ec = Kg \quad (18)$$

The changes in the internal energy densities are

$$\underline{e}_g = h_g s - g^2\Lambda(T_g) + bT_g s\eta/\tau + Ecb\tilde{T}_c - KgbT_g \quad (19)$$

with the abbreviation $b = 1.5k_{\rm B}/\mu_p = 0.013$ in our units. The first term is the heating of the gas by massive stars, e.g. by photoionization. One assumes that all stellar ionizing photons are absorbed by the gas. Similarly, the deposition of mechanical energy is described in the same way, if all energy is absorbed. Photoionization with an efficiency of 10^{-4} for the conversion into thermal energy (cf. Köppen et al. 1995) yields: $h = 21900$. The second term is radiative cooling with a general cooling function $\Lambda(T)$ (cf. Theis et al. 1992). In our units, and by noting the use of mass densities instead of number densities:

$$\Lambda(T) = 1800\ T^{0.5} \quad (20)$$

The third term represents the energy gained by the mixing of the stellar ejecta. For simplicity, one assumes that the ejecta have the same temperature as the gas. As this term is usually quite small, this assumption is not important. The last terms are the energy gained by the gas through the addition of the evaporated cloud material — whose temperature shall be $\tilde{T}_c$ — and the energy lost by the gas that is condensed onto the clouds. Likewise, the energy density of the cloudy medium is changed

$$\underline{e}_c = h_c s - c^2\Lambda(T_c) - \Psi bT_c - EcbT_c + KgbT_g \quad (21)$$

by gains from (photoionization) heating by massive stars and losses by radiative cooling, by the losses due to matter locked up into stars (usually a minor term), and by the losses due to evaporating material or by gains from incorporating the condensing gas. The latter temperature is designated as $\tilde{T}_g$. For simplicity, we consider $h_c = h_g$.

2.2. Experience from full numerical solutions

This system of equations had been incorporated into our chemodynamical models in various forms, e.g. to compute the evolution in a closed box system (Hensler & Burkert 1991, Theis et al. 1992), and coupled with 1-dimensional hydrodynamics to study the evolution of spheroidal systems (Theis et al. 1992). Star formation was found to follow two modes: in a continuous way as the consequence of self-regulation (cf. Köppen et al. 1995), but in a certain range of densities it would fluctuate very strongly. These fluctuations are actually regular, large amplitude, non-linear oscillations of the cloud temperature and hence of the star formation rate (cf. Fig. 7 in Theis et al. 1992). The period of the order of 100 Myrs is longer than the cooling time scale of the cloud gas. This indicates that the strongly damped oscillations which may occur before star formation reaches the self-regulated mode (Köppen et al. 1995) cannot be the origin. Further inspection showed that the oscillations showed up most strongly in the cloud temperature and the gas density, rather weakly in the gas temperature, and were almost absent in the cloud density.

In Fig. 1 we show the track in the $(g - T_c)$-plane of a model entering the oscillatory mode and performing a few cycles. For the last time point, Fig. 2 depicts the streamlines of the full system of equations, projected into the $(g - T_c)$-plane. In both figures, the dashed curves indicate the border where the system changes between evaporation (at lower densities) and condensation. The streamlines in the evaporation regime are always oriented towards increasing gas density, and bunch together to form a funnel centered at constant $T_c \approx 2400$ K, which is marked in Fig. 1. Along this feature, the system evolves until the gas density exceeds a critical value, and condensation sets in. In the condensation regime, the temperature increases, until

another funnel is reached. Marked with a dot-dashed line, it can be seen in the upper centre, curving down and towards lower values of g. At $\lg(g) = -1.9$ and $T_c = 5000$ K, it meets the condition where the system is switched back to evaporation, where it again starts its evolution towards the evaporation funnel, the cycle is repeated. In the numerical solution, one notes that the condensation funnel is at a different position from the one shown in the streamline picture. This is because during the cycle, the stellar density and the gas temperature do change, and thus Fig. 2 refers only to the state of the system just before it enters the condensation regime.

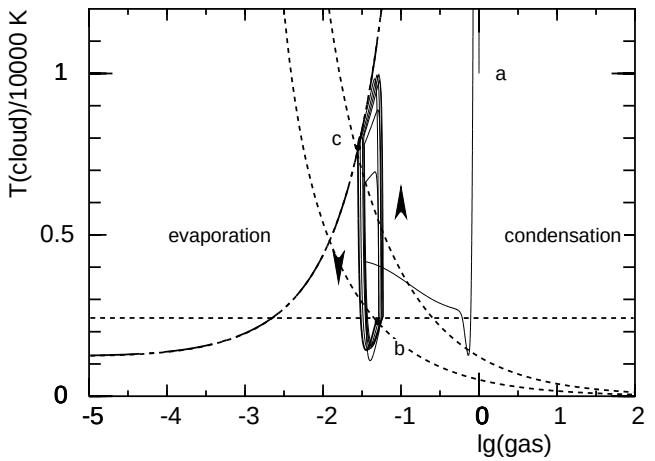

Fig. 1. Evolution of the full system until the start of the oscillations (solid line). The dashed lines descending to the right are the loci where the system switches from evaporation and condensation (lower curve) and vice versa (upper curve). The horizontal dashed line is the locus of the evaporation funnel and the dot-dashed curve depicts the condensation funnel (see text).

3. The 'inner' system

The fact that the numerical simulations show the appearance of the oscillations primarily in cloud temperature and gas density indicates that these two variables form an 'inner' system whose behaviour is governed by the values of the control parameters (c, T_g). In the following, we concentrate on the characteristics of this inner system. Rather than discussing the full set of equations with the prescriptions and rate coefficients from McKee & Begelman (1990) or Cowie et al. (1981), it is useful and quite sufficient a more simple configuration, and to assume that the rate coefficients for condensation and evaporation are constants (K, E). For the criterion of the switching between condensation and evaporation, we retain the prescription to compute σ_0 as given above, but we shall consider the threshold value of 0.03 as another free constant, denoted by σ.

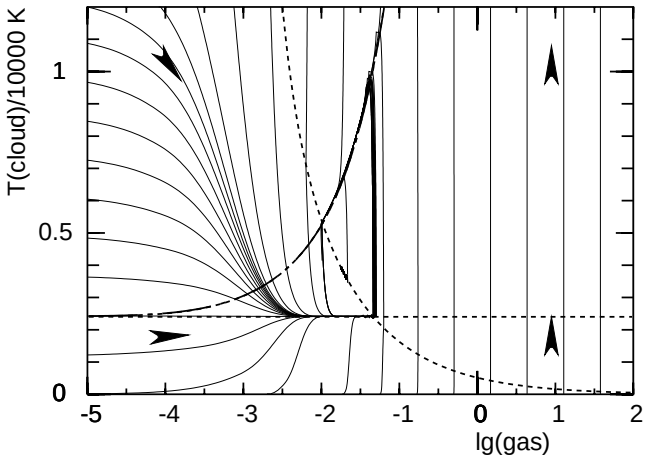

Fig. 2. The streamlines of the full system for the conditions shortly before it would switch from evaporation to condensation (cf. Fig. 1).

The streamlines shown are projections into the g, T_c space, with specified values for c, s, and T_g. Therefore we investigate the behaviour of the following system, which is split up into the case for evaporation:

$$\dot{g} = s\eta/\tau + Ec \tag{22}$$
$$\dot{e}_c = h_c s - c^2\Lambda(T_c) - \Psi b T_c - Ecb T_c \tag{23}$$
$$\dot{T}_c = (h_c s - c^2\Lambda(T_c))/(bc) \tag{24}$$

and condensation:

$$\dot{g} = s\eta/\tau - Kg \tag{25}$$
$$\dot{e}_c = h_c s - c^2\Lambda(T_c) - \Psi b T_c + Kgb\tilde{T}_g \tag{26}$$
$$\dot{T}_c = (h_c s - c^2\Lambda(T_c))/(bc) + Kg(\tilde{T}_g - T_c) \tag{27}$$

where we used $\dot{e}_c = b(\dot{c}T_c + c\dot{T}_c)$, and we take note that c may also change. It is not necessary to consider the evolution of the low-mass stars and the remnants, as they have no interaction with any of the gas phases, and we do not include the influence of the gravitational field.

3.1. Pure evaporation

Figure 3 shows the streamlines of the simplified system, for evaporation ($\sigma = 0$), which are described by:

$$\frac{dT_c}{d\ln g} = g\frac{\dot{T}_c}{\dot{g}} = \frac{g(h_c s - c^2\Lambda)}{bc(s\eta/\tau + Ec)} \tag{28}$$
$$= \frac{g}{bc^2}\frac{h_c(s/c^2) - \Lambda}{c\eta(s/c^2)/\tau + E} \tag{29}$$

which show the following features:

– a horizontal streamline exists where there is a balance between stellar heating and radiative cooling $h_c(s/c^2) = \Lambda$,

marked by the dashed line. Along this evaporation funnel, the evolution proceeds with the clouds being essentially in thermal equilibrium. The resultant cloud temperature is lower for:
  - low stellar density s/c^2
  - low heating rate coefficient h_c
  - higher cooling efficiency
- at $g = 0$ all streamlines are horizontal, which is merely a feature of the logarithmic representation and gives the impression of the fan structure on the left as the lines bunch together into the funnel. Since the slopes generally increase for large g/c^2, the extend of of this region is limited to lower gas densities for
  - low stellar density s/c^2
  - low cloud denmsity c
  - low evaporation rate coefficient E

It is evident that regardless of its initial state, the system will always end up in the funnel where it evolves at constant cloud temperature towards higher gas densities.

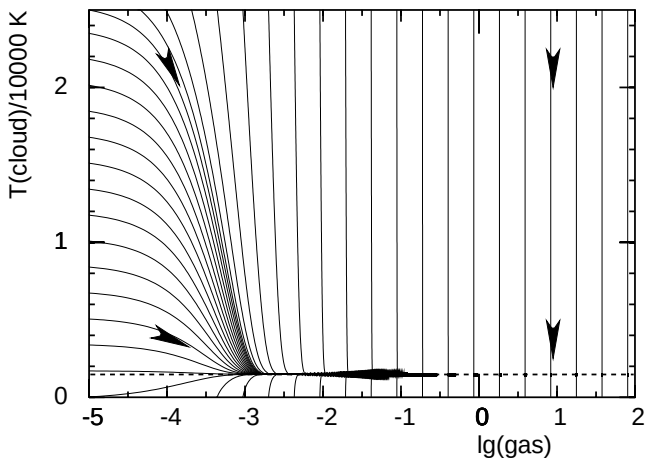

Fig. 3. Streamlines in the evaporation regime for the simplified model. The position of the evaporation funnel is marked by a dashed line.

3.2. Pure condensation

When condensation occurs ($\sigma = \infty$), the streamlines show a more complex structure:

$$\frac{dT_c}{d\ln g} = \frac{g(h_c s - c^2\Lambda) - bKg(\tilde{T}_g - T_c)}{bc(s\eta/\tau - Kg)} \quad (30)$$

$$= \frac{g}{bc}\frac{h_c(s/c^2) - \Lambda + b(gK/c^2)(\tilde{T}_g - T_c)}{\eta(s/c^2)/\tau - (gK/c^2)} \quad (31)$$

as shown in Fig. 4. Also, in Fig. 5, we consider a model with a lower value of $\tilde{T}_g = 20\,000$ K. Apart from the region of horizontal streamlines at $g = 0$, as before, the pattern has these features

- at low gas densities, a horizontal streamline exists where $h_c s = c^2\Lambda$, the same condition as for the evaporation funnel. Its position agrees closely with the upper dotted curve, which is the locus where one has thermal balance in the clouds $e_c = 0$.
- at large gas densities, a horizontal streamline appears where $T_c = \tilde{T}_g$. The lower dotted curves shows where the cloud temperature does not change with time. The clouds are not in thermal equilibrium, as the condensation occurs faster than the cooling time, and the clouds are kept at the condensates' temperature.
- these two parts are connected with a curve, along which the evolution proceeds towards thermal equilibrium in the clouds. Where the transition occurs, is essentially determined by the ratio of Λ and $b(gK/c^2)(\tilde{T}_g - T_c)$, so it takes place at lower gas densities for
  - high condensation rate coefficient K
  - high condensates' temperature $\tilde{T}_g$
  - high cloud densities c

There is an additional feature: the intersection of the condensation funnel with the condition $s\eta/\tau - Kg = g_* = 0$ at $\lg g = -2.6$ and $T_c = 2500$ K (in Fig. 4) forms an attracting node. In Fig. 5 it is more evident that it occurs on the branch where the temperature is constant at low densities. The evolution od the clouds proceeds in thermal equilibrium and a balance between stellar gas return and condensation. This point is found at low gas densities, if s is low or the condensation coefficient K is large.

Independent of the initial conditions, the system always reaches the condensation funnel, where the clouds cool and the gas density decreases. But the system may get stuck and enter an equilibrium evolution at constant gas density.

3.3. Overall evolution

Figure 6 shows the streamlines for the complete simplified system. The two regimes of pure evaporation and condensation are separated by the curve where the criterion of McKee & Cowie (1977) is met exactly ($\sigma_0 = \sigma = 0.03$). How the position of this switching line depends on the parameters, can be worked out from the prescriptions (Eqns.6 to 11).

For a cloud filling factor smaller than unity, which is the common case, one obtains:

$$\sigma_0 \propto T_g^2 R_{\rm cloud}^{-1} g^{-1} \propto T_g^2 T_c^{0.5} g^{-0.5} \quad (32)$$

which means that the curve $\sigma_0 = \sigma$ is independent of the cloud density c, but it moves towards lower gas densities, if one raises the threshold σ, or lowers the gas temperature T_g which is the most sensitive parameter.

Under the conditions chosen for the figure, (c, s, and T_g), the two funnels meet the switch line in a way suitable for the system to evolve in a closed cycle. When condensation starts, the clouds are rapidly heated up by the gas condensates, until the

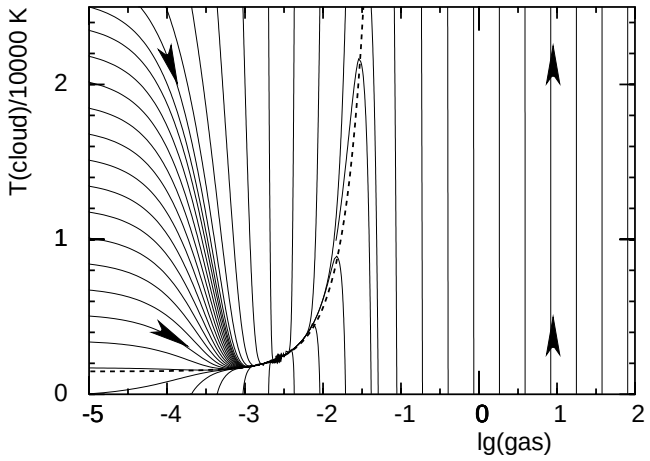

Fig. 4. Streamlines of the simplified model in the condensation regime. The temperature of the condensate is assumed to be equal to the gas temperature. The dot-dashed line indicates the locus for thermal equilibrium in the clouds.

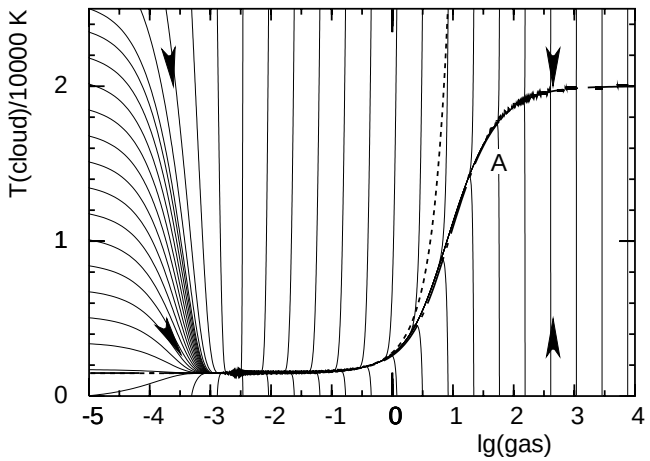

Fig. 5. Same as Fig. 4, but for a temperature of the condensate $\tilde{T}_g$ = 20/, 000 K. The dot-dashed lines show where thermal equilibrium in the clouds exists (left), and where the cloud temperature is constant (right).

condensation funnel is reached. There, the clouds cool radiatively, evolving towards thermal equilibrium. But even before this is reached, the criterion is met from the other regime, and evaporation commences. Lacking the heating by the hot condensing gas, the clouds cool until the cooling rate is balanced by the stellar heating rate only. The on-going evaporation of the clouds increases the gas density, until the condensation criterion is again fulfilled.

This cycle is driven by the deposition of thermal energy of the condensates into the clouds, which radiate it away in the form of line emission. Thus the source which makes the cycle possible is the deposition of sufficiently hot material onto the clouds.

If the condensates do not change the temperature of the clouds ($\tilde{T}_g = T_c$), the condensation funnel would be merely a horizontal line at the same height of the evaporation funnel. Thus the intersection of this line with the switching curve is an attracting node, into which the system will always come, and where it evolves obeying these conditions:

$$h_c s = c^2 \Lambda(T_c) \quad (33)$$

$$\sigma = \sigma_0 \propto T_g^2 T_c^{0.5} g^{-0.5} \quad (34)$$

$$Ec = Kg \quad (35)$$

$$(36)$$

which are thermal equilibrium of the cloud gas, the locus on the switch line, and Eq. 18.

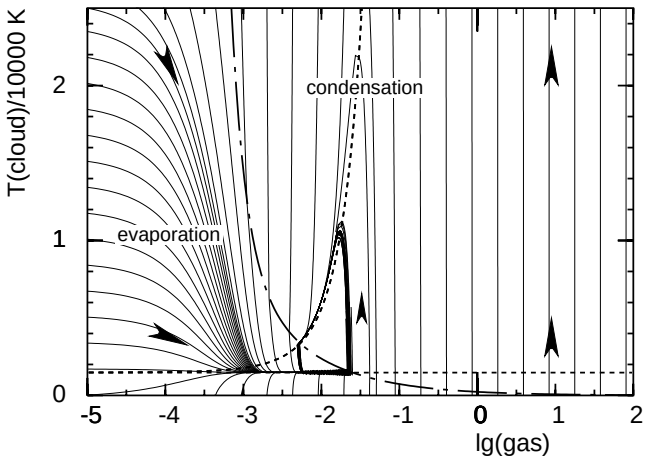

Fig. 6. The overall behaviour of the simplified system: streamlines for the combined equations. The dashed curves are the loci where the clouds change from condensation to evaporation.

4. Conditions for oscillations

The existence of the funnels makes it relatively easy to predict the conditions under which oscillations occur, i.e. where the system follows a closed path in the space. We must merely note the relative positions of the funnels and the switch line. From the pattern of the streamlines it is clear that a condition for the occurrance of oscillations. is that the evaporation funnel should intersect the switching line at a lower cloud temperature (or higher gas density) than the condensation funnel.

Let us first discuss qualitatatively when oscillations can occur and how this depends on the parameters: As will be shown below, the two intersection should be rather close together, to have periods smaller than the lifetime of the galaxy. Thus, the

cloud temperatures remain close to the values of the evaporation funnelm i.e. a few thousands K. The gas density where the switch line occurs, increases with increasing cloud density, but decreases quite strongly with gas temperature. If the gas has time to reach thermal equilibrium, T_g grows with larger stellar density. Thus, a small value of s places the switch line at lower values of g. The pattern of the streamlines indicates that oscillations are favoured, if at the gas densities under consideration the evaporation funnel is low, the condensation funnel high, and that the intersections do not occur at so low a density that the streamlines are horizontal. From the earlier discussion of the dependence of the streamline pattern on the parameters one concludes: The low evaporation funnel is favoured by a low stellar heating rate (either s/c^2 or h_c being low), and the avoidance of $g = 0$ (by low values of E, s/c^2, and c). This means that for too large a cloud density, the streamlines in the evaporation regime become so flat that they do not reach the funnel. Thus this poses an upper limit in c for the occurrance of oscillations. The high condensation funnel is favoured by large $\tilde{T}_g$, large K, small h_c, and small s/c^2; avoiding streamlines too flat to reach the funnel favours large cloud densities. Therefore there is a lower limit for oscillations.

4.1. Dependence on control variables

More quantitatively, one notes that the positions of the funnels and the switching line are determined by the c, s, and T_g. While one may accept c as characterizing the actual or initial state of the gas-star system, the stellar density depends on the SF history, and the gas temperature would be influenced by the current number of stars, but because of the (usually) much lower density, the cooling time scales in the gas would be long, and it is not appropriate to assume thermal equilibrium in the gas.

If one makes the assumption that the stellar density is constant $\dot{s} = 0$, one gets $s = \xi\tau\Psi(c, T_c)$ which is the low density limit of our model with one gas phase. Then it is possible to judge the behaviour of the system.

1. The condition $\sigma_0(c, g, T_c, T_g) = \sigma$ is a non-linear equation for T_c as a function of the other variables. Numerical solution gives the switching curves shown in the preceding figures.
2. The evaporation funnel $\dot{T}_c = 0$ gives $h_c\xi\tau\Psi(c, T_c) = c^2\Lambda(T_c)$ which together with the specific form of the SBF Eq. (....), constitutes a non-linear equation for T_c as a function of c. Note that it does not depend on g, and so it gives a constant line in Fig. 3, but it is also independent of T_g and hence any further assumptions.
3. The condensation funnel can well be characterized by the $\dot{T}_c = 0$, i.e. $h_c\xi\tau\Psi(c, T_c) + Kg(\tilde{T}_g - T_c) = c^2\Lambda(T_c)$ which together with the SBF yields an equation for T_c as a function of c, g, and T_c.
4. The system enters the condensation regime at the intersection of the switching line and the evaporation funnel, which is easily computed by finding the gas density $g = g_{\rm ON}$, where the cloud temperatures from above conditions (1) and (2) are equal and yield: $T_{\rm ON}$.
5. Condensation stops at the intersection of (1) and (3), which gives $g_{\rm OFF}$ and $T_{\rm OFF}$.

All these conditions can be solved uniquely and easily by numerical means, such as the bisection method, though some care has to be taken to start the nested iterations with suitable values. This can be done for any pair of values of c and T_g.

The positions of the two points labeled ON and OFF are quite sufficient to characterize the system's behaviour:

1. If $g_{\rm ON} > g_{\rm OFF}$, oscillations *may* occur, as shown in Fig. 6. However, whether a complete cyclic path is formed, depends on the slope of the streamlines in the regime which is being entered:
   - At the ON point, the streamlines in the condensation regime should not be so flat that the condensation funnel is not reached. When this occurs, the system reaches some point on the switching line between ON and OFF points. Flat streamlines are found in the low-density part, as seen in Fig. 4.
   - At the OFF point, the streamlines in the evaporation regime must be steep enough to reach the funnel. Otherwise, the system comes to rest at some intermediate point, too. As Fig. 3 shows this may occur in the low-density part.
2. If the two points coincide, or are very close to each other, the system will come to rest at this intersection of the funnels and the switching line.
3. The remaining case $g_{\rm ON} < g_{\rm OFF}$ is not possible: because the stellar density was assumed to be constant, the condensation and evaporation funnels coincide at low g. In the full system, this can be different (cf. Fig. 1).
4. If the condition $\dot{g} = 0$ occurs in the condensation regime, the system will come to 'rest' at this node. The cloud matter is converted into stars, while the stellar ejecta balance the condensation rate.

In Fig. 7 we show in the $c - T_g$-plane the regions of the different behaviour, amrked by the different symbols. There exists a rather large region where oscillations are possible. This includes the cloud density of $\lg(c) = 0$, which is the model shown in Fig. 6.

The knowledge of the crossing points makes it also possible to estimate the amplitude of the temperature variation, the times spent in the evaporation and condensation phase which give not only the period of the cycle, but also the mark-space-ratio for the star-formation bursts.

Having determined the two crossing points in the g, T_c-plane, we can not only judge from their relative positions, whether oscillations occur, but we can also estimate the period and the mark-space-ratio of the pulses in temperature. The evolution during evaporation (from OFF to ON points) can roughly be estimated by $\dot{g} \approx -Ec$ the time for the evaporation phase:

$$\Delta t_{\rm evap} \approx \frac{g_{\rm ON} - g_{\rm OFF}}{Ec} \tag{37}$$

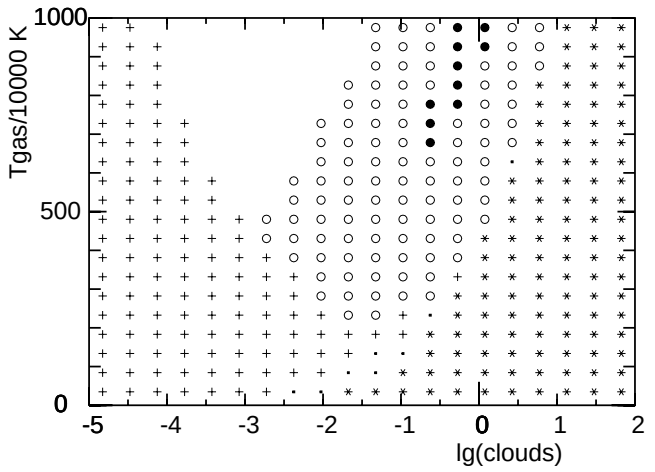

Fig. 7. The behaviour of the simplified system as a function of the control parameters c and T_g, as estimated with the assumption of constant stellar density: Circles denote where ocsillations occur (filled circles: the gas temperature is within a factor of 2 of the value for thermal equilibrium of the gas). In the white area, oscillations also occur, but with a period of over 5 Gyrs. Plus-signs indicate where the streamlines are to flat, and the system comes to rest at an intermediate point on the switching line. Small dots show that the funnels meet very closely, and the asterisks indicate that the system enters the $g = 0$ equilibrium.

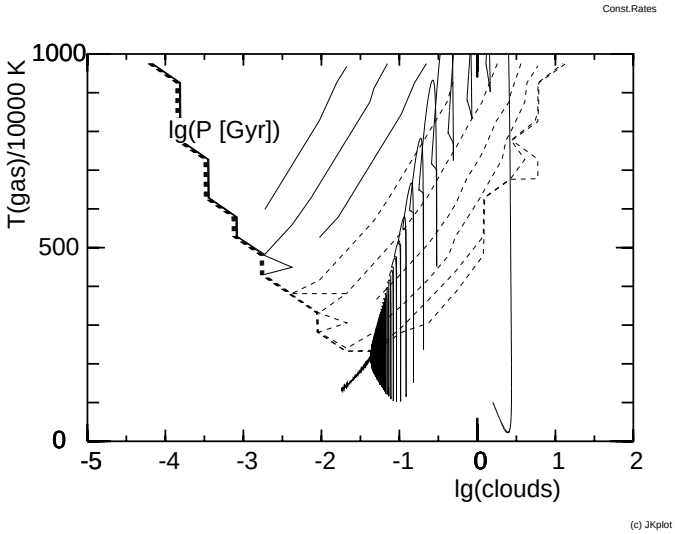

Fig. 8. The oscillatory region in the $c, \lg(T_g)$ diagram: Lines of equal period (10, 3, and 1 Gyr (solid), and 0.3 .... 0.001 Gyr (dashed)). Also shown is the computed evolution of the system ■

Similarly, the time for the condensation phase $\Delta t_{\rm cond}$ is obtained from $\dot{g} \approx -Kg$:

$$\Delta t_{\rm cond} \approx \ln g_{\rm OFF}/g_{\rm ON}/K \tag{38}$$

One notes that when the points are far apart, the amplitude of the oscillation is large, but so do the times and the period: large oscillations could take longer than the evolution of the galaxy, and thus are of little importance. How much of the period is spend in the condensation regime, depends mainly on the ratio K/E of the rate coefficients.

In Fig. 8 we show the lines of equal period $P = \Delta t_{\rm evap} + \Delta t_{\rm cond}$ (where both times are positive) in the space of the control variables c, T_g. For comparison we also show the track of a evolving system. One notes that at first, after the system enters the oscillatory region at high gas temperatures and starts with quite long periods (100 Myrs), which become shorter as time passes. Finally, the track leaves the oscillatory region at $\lg(g) = -1.4$ and enters the ■ condensation mode. Note that this occurs exactly as predicted by the analysis of the switching points (cf. Fig. 7.

In Fig. 9 we show the ratio of the times spend the condensation and evaporation regimes. Note that this ratio is just a function of the period ■ simple argument why ■ In the model whose evolution was depicted in Fig. 8 the period becomes shorter with time, and so the fraction of the time spent in the condensation phase increases, where the cloud temperature is large. So, initially the clouds have higher temperatures, and hence lower star formation rates, during relatively short phases, which become longer as the evolution progresses. Thus the averaged star formation rate is gradually decreased.

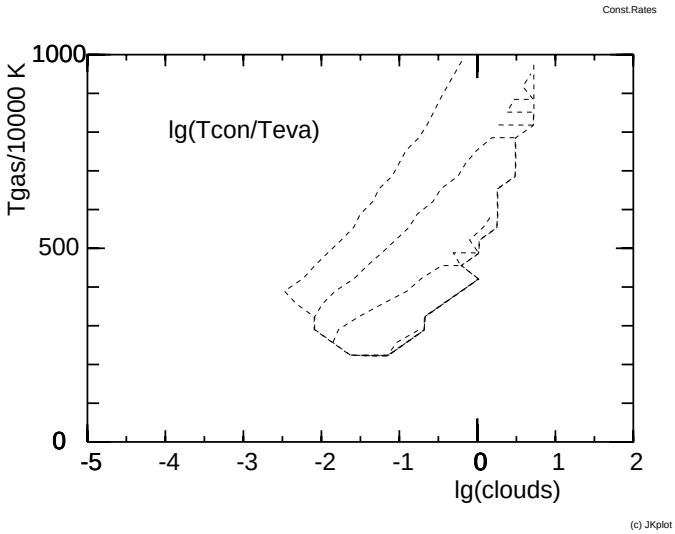

Fig. 9. The ratio of the times spend in the condensation and evaporation regimes: shown are lines of constant ratio – which go from lower left to upper right, for the ratio 0.01, 0.03, 0.1, and 0.3, increasing towards lower values of T_g. ■ this figure is NOT final

Some remarks that the solutions shown and the streamline patterns shown are representative. I've never seen anything completely different in structure. Why that? Can I give the reason that or why I have seen the whole behaviour space?

This picture gives some impression where oscillations may occur, but it cannot predict whether they do occur as part of a complete time evolution from some initial starting state..

NOTE: if $\tilde{T}_g = T_c$ one always gets a single dot, where the both funnels intersect the switch line in the same point. Evidently this is an attracting node.

NOTE: finite stellar lifetime: the small wiggle in T_c in the figure of Theis et al. is when the system settles into evaporation funnel solution.

NOTE: point out that the oscillations in temperature cause oscillations in the SFR. emphasize that these are NOT bursts of star formation in the sense of enhanced SF activity, but that the self-regulated SFR is periodically cut-off during the condensation phase. Thus we have interrupted 'normal' SF.

4.2. Dependence on the parameters

Though the values for the modeling parameters such rate coefficients had been derived from the results of other physical models (i.e Cowie et al. ■). There are several idealizations incorporated in these models and further simplifications that are used to define our system of equations (geometry of the clouds, criterion for the mass of a cloud, neglect of the proper density profile within a cloud, etc.). Therefore, the adopted values and parametrizations must not be taken at face value, but is is evident that they could be still rather different from the true situation in the interstellar medium. How sensitive is the evolution of the system to changes of the parameter values?

The rate coefficients for evaporation and condensation: For the simplified model we had used constant values. If one uses the proper expressions given by Cowie et al. (198■), the exact shape of the streamlines changes, and the details of the evolution, but as one can directly see from a comparison of Figs. 2 and 6 the basic structure does not change. This is also our experience from computations of the time evolutions.

As far as the value of these coefficients is concerned, their effect can be described by their influence on the streamline pattern: a decrease of E makes the slopes in the evaporation region steeper, thus making it easier to reach the funnel, and thus the condition for oscillations. This also prolonges the evaporative phase, i.e. when the system is in the mode of self-regulated star-formation. An increase of K brings the intermediate part of the condensation funnel towards lower densities, which separates the ON and OFF points, and favours oscillations. This shortens the time spend in the condensation regime, and the time during which star-formation is suppressed.

The rate coefficients h_g, h_c for heating of the gas or clouds by stellar photons can also increase the likelihood for oscillations: Decreasing h_g will decrease T_g which moves the switch line towards higher gas densities, away from the region with flat streamlines. This can be countered by the decrease of $\tilde{T}_g$. Decreasing h_c lowers the evaporation funnel.

The most critical parameter is the temperature of the condensates $\tilde{T}_g$: large values, such as T_g, are vital for the occurrance of oscillations. Numerical computations show that sometimes a fairly slight reduction of $\tilde{T}_g$ below the gas temperature — by 30 percent — suffice to make all oscillations disappear. If one allows the condensate to cool down to 20 000 or 10 000 K, no oscillations are seen. The case $\tilde{T}_g = T_c$ will be considered in the next section.

5. The physically consistent solution

In the derivations, McKee & Cowie (1977) and McKee & Begelman (1990) assume a stationary solution for the gas flow between cloud and intercloud matter, with a fixed temperature profile. This means that any parcel of condensing gas follows the local temperature from the intercloud gas to the cloud interior, and that it arrives in the cloud at the cloud's temperature: $\tilde{T}_g = T_c$.

As shown in Sect. 3.3, the system then evolves in thermal equilibrium of the cloud gas, on the switching line $\sigma_0 = 0.03$, and with $Ec = Kg$. This last condition implies that the evaporation and condensation terms in Eqns. 1, 2, 19, and 21, and that thermal equilibrium is equivalent to constant temperature. Under normal conditions, one finds that $g \ll c$, so that one has $c \approx c + g$ and the evolution is described by

$$\begin{aligned} \frac{d}{dt}(c+g) &= -\Psi + s\eta/\tau \\ \dot{s} &= \xi\Psi - s/\tau \\ \dot{r} &= (1-\xi)\Psi + (1-\eta)s/\tau \\ e_c &= h_c s - (c+g)^2\Lambda(T_c) - \Psi b T_c \end{aligned}$$

This is nothing but the single-gas-phase system which has been shown to evolve almost exclusively in an equilibrium between star formation and its inhibition due to the heating of the ambient gas by the massive stars (Köppen et al. 1995).

etc.....

Acknowledgements: We thank M. Samland and R. Spurzem for valuable discussions. J.K. thanks the Deutsche Forschungsgemeinschaft for financial support (project He 1487/13-1) and the Observatoire de Strasbourg for the hospitality.

References

Cowie L.L., McKee C.F., Ostriker J.P., 1981, ApJ **247**, 908
Hensler G., Burkert A., 1991, in: Proc. Elba Workshop "Chemical and Dynamical Evolution of Galaxies" eds. F.Matteucci et al., p.168
Köppen J., Theis Ch., Hensler, G., 1995, A&A **296**, 99
McKee C.F., Begelman M.C., 1990, ApJ **358**, 392
McKee C.F., Cowie L.L., 1977, ApJ **215**, 213
Samland M., Hensler G., Theis Ch., 1996, ApJ , submitted
Theis Ch., Burkert A., Hensler G., 1992, A&A **265**, 465

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.