Astrophysics with the Computer: Ram pressure stripping of dsik galaxies

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1 Astrophysics

In galaxies clusters one finds among normal elliptical and spiral galaxies also a few spiral galaxies which have a smaller mass in neutral hydrogen gas than normal spirals. Also one observes that their HI disks are smaller than the stellar disk. The current idea is that these galaxies have been stripped of their outer HI disks by the ram pressure acting on the galaxy as it travels through the hot intracluster gas. From X ray observations it is known that this gas is most dense in the centre of the cluster. Therefore it seems likely that a galaxy on an orbit which talkes it close to the centre or right through the centre will experience a ram pressure which acts on the gas disk. This pressure will accelerate a gas element. If this force is strong enough and lasts long enough, the gas element can reach escape velocity and is thus removed from the galaxy.

There exist rather sophisticated simulations of this interaction of the gas disk of the galaxy with the intracluster medium. The detailed treatment is rather complicated as it should ideally be done in 3 dimensions, following the galaxy on its trajectory through the cluster as well as treating the hydrodynamics of the gas-gas interaction. It is only nowadays that such complete simulations can be done, and they are being done.

One simple argument has been used for a long time to estimate when a gas element is removed from a galaxy. This criterion, by Gunn & Gott (1971; ApJ 176, 1), says that if the maximum ram pressure exceeds the maximum of the gravitational force which holds the gas element bound to the disk, then the element is removed. This criterion only needs the maximum ram pressure ever experienced by the gas element, but it does not say how long the pressure needs to have acted. But it seems quite clear that if a small pressure acts a long time, it could also remove the gas. Or if a galaxy passes through a small cluster, then the pressure can act only brfiefly, and one would expect that the gas cannot be removed.

In this project we shall consider a very simple model for this process: Imagine a point mass, bound in a potential, which is subjected to some force during some interval of time. Let us find out under which conditions for this force impulse the point mass is accelerated up to escape velocity. This will give some idea of what happens to a galaxy traveling through the gas in a cluster. And it will permit us to check when the Gunn & Gott criterion breaks down.

2 The Equations

We consider the vertical motion of a point mass m in a given and fixed potential well Φ:

$$\Phi(z) = -\Phi_0 \exp(-(\frac{z}{H})^2) \tag{1}$$

with some scale height H which measures the thickness of the disk. You might work in normalised coordinates $z' = z/H$, so that you measure velocities also in these units, but not in say: km/s. We may use any other form of potential, as long as it has a single, stable minimum at $z = 0$ and tends to zero for large distances. The equation of motion is simply

$$m\frac{d^2z}{dt^2} = -m\frac{d\Phi}{dz} + F(t) \tag{2}$$

where $F(t)$ is the time dependent external force. We may model this force pulse like the ones a galaxy would experience: as the galaxy approaches the cluster centre the force increases steadily; it reaches a maximum when the galaxy reaches the centre, and then the force decreases in the same way as the galaxy moves away. We may take something like a Gauss function:

$$F(t) = F_{\max} \exp(-(\frac{t - t_0}{\tau})^2) \tag{3}$$

with the time t_0 when the galaxy passes the centre. This assumes that the density profile of the gas in the cluster is symmetrical, and also that it is rather smooth. We shall neglect the possible presence of clumps, denser regions, or dense lanes in the gas ...

Solution of the equation of motion requires the specification of the initial position and velocity of the body. For simplicity, we shall assume that initially the body is at rest in the galactic plane, i.e. $z(0) = 0$ and $v(0) = 0$. But you should also investigate what happens when you take other assumptions, since gas clouds do not need to be in the galactic plane and they may have non-zero speeds.

In the context of our simple model, the criterion of Gunn & Gott can be formulated as: If

$$F(t)|_{\max} \geq m|\frac{d\Phi}{dz}|_{\max} \tag{4}$$

then the gas element can escape. Here, one needs to have the maximum value of the restoring force $-d\Phi/dz$ which will occur at some height above the galactic plane.

3 Numerical Methods

The equation of motion is solved in this fashion: The net force at any instant of time t gives us the acceleration

$$a(t) = \frac{d^2z}{dt^2} = -\frac{d\Phi}{dz} + \frac{F(t)}{m} \tag{5}$$

You see that for convenience it will be better to measure the force F in terms of F/m so that we do not need to bother about the actual mass of the body. To do the integration of time, we take a constant time step Δt, during which we assume that the acceleration at the 'old' time t is constant. Thus the velocity of the body will change during this time step:

$$v(t + \Delta t) = v(t) + a(t)\Delta t \tag{6}$$

Of course, during this time interval, the acceleration changes, as the mass point flies to another position. So what one really needs for the acceleration is a suitable mean value for each time step. But in order to compute this, we need to know the new position, ... which is what we are just going to compute. So, such a more accurate method (which is called *implicit* since it uses information about the new position) needs an iteration for each time step and it is more complex to program. For the time being, we shall use the simpler method, but please remember, it is less accurate. It is called an *explicit* method, as it uses only information from the old time.

To compute the new position of the mass point at the new time $t + \Delta t$ we assume that the velocity is constant. However, the same question as before is raised: which velocity shall we use? We have two: the old one $v(t)$, or the new one $v(t + \Delta t)$, and we could even take some average value. What do you think? If we use the old velocity, the new position is

$$z(t + \Delta t) = z(t) + v(t)\Delta t \tag{7}$$

These computations are done for all the points, and after all the new positions are ready, one repeats the operation for the next time time step ...

4 What to do

First of all, write the program so that you can follow the movement of the body as a function of time. You need to check that it computes the correct solution. We do not know the correct solution for this particular potential and with some external force pulse. So better consider a simpler case where you know the exact solution, for instance the harmonic oscillator without an external force: So you have to put in the potential for that case:

$$\Phi(z) = -\Phi_0 + (\frac{z}{H})^2 \tag{8}$$

for example. Then you can get the analytical solution of $z(t)$ and compare with the program's output. The oscillations should have the same period. Another way of checking is to plot the trajectory in phase space (z, v) which for the harmonic oscillator has a certain simple shape ...

Now you can start exploring the real problem. It is quite interesting to see how this body moves in this non-harmonic potential well! Also the trajectories in phase space.

The body will escape, if its total energy has reached a non-negative value:

$$\frac{E}{m} = v^2 + \Phi(z(t)) \geq 0 \tag{9}$$

I suggest you play around with the parameters of the force pulse to get a feeling of when the body can be removed from the potential well. Just one suggestion: better use a time t_0 for maximum force so that at the start of the calculations the force is still negligibly small ... otherwise, the body receives a sudden kick right at the beginning which might cause problems.

Next, you should make a systematic study of where in the space of the parameters $F_{\max}$ and τ the body escapes, such as plotting a curve in this coordinate system while scanning through the parameters. Also, you can plot where in that diagram the Gunn & Gott criterion is fullfilled.

Can you find a simple condition or formula from which we could estimate when a force impulse is suffiencient to kick the body out of the well? Is this related to the total momentum

$$\int F/m dt \tag{10}$$

up to the point of escape or the momentum transferred to the body

$$\int (F/m - d\Phi/dz) dt \tag{11}$$

or should one integrate only for times when $F/m > d\Phi/dz$ ...

So, does the Gunn & Gott criterion underestimate the removal of gas from galaxies, or does it overestimate this?

How do things change when you use different shapes of the potential, different depths Φ_0?