Evolution of Galaxies:

Chemical Evolution: The Simple Model

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http://astro.u-strasbg.fr/~koppen/JKHome.html

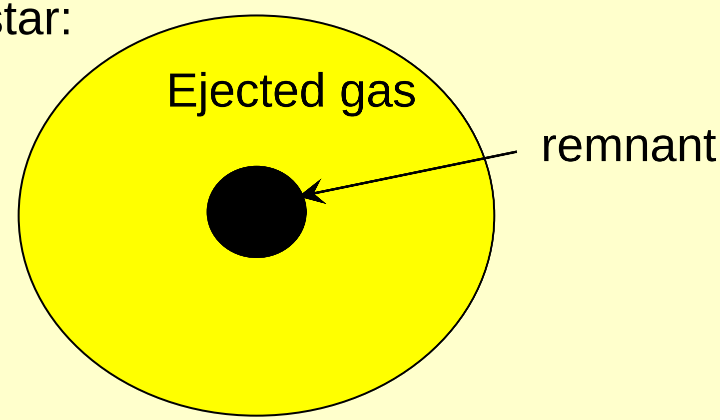
Assumptions: e.g. solar neighbourhood

- Closed box (no mass exchange): **Simple Model**
- volume large enough (100 pc) and time interval long enough (0.1 Gyr) to define average values
- (solar nh'd = solar cylinder: integrate over height above Galactic plane and azimuth – neglect vertical kinematics and rotation)
- Gas always well mixed (why not? Makes life easier)
- Constant IMF (why not?)
- Initially 100% gas, metal-free (why not?)

Gas mass equation

$$\frac{dg}{dt} = -\text{star formation} + \text{gas ejected by stars}$$
$$= -\Psi(t) + \int_{m_{\min}(t)}^{m_{\max}} E(m) \cdot \Phi(m) \cdot \Psi(t - \tau_{\mathrm{MS}}(m))\, dm$$

a star:



$\tau_{MS}(m)$ = lifetime of stars of initial mass m

≈ time on main sequence

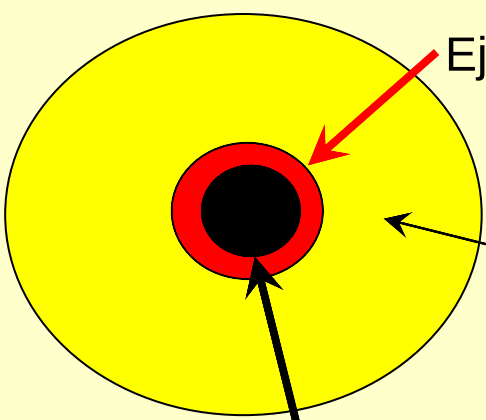
Metal mass equation

$$\frac{dgZ_i}{dt} = -Z_i\Psi(t) + \int_{m_{\min}(t)}^{m_{\max}} E_i(m)\cdot\Phi(m)\cdot\Psi(t-\tau_{\rm MS}(m))dm$$

Ejected fractional stellar mass in the form of element *i*

$$E_i(m) = \quad p_i(m) + Z_i(t-\tau_{\rm MS}(m))\cdot E(m)$$

Ejected freshly produced metals



Ejected unprocessed gas, with ‘old’ metals

Old and new metals locked up in remnant

Instantaneous Recycling Approximation

- metals are produced by massive stars (> 10 $M_\odot$) which live only <10 Myrs
- they also eject most of the gas
- the numerous long-lived stars do not eject metals (O, Ne, Ar, S, …)

- ➔ neglect stellar lifetimes in the ejecta

$$\int_{m_{\min}(t)}^{m_{\max}} E(m)\Phi(m)\Psi(t-\tau_{\rm MS}(m))dm \approx \Psi(t)\int_{m_{\min}(t)}^{m_{\max}} E(m)\Phi(m)dm$$
$$=: \Psi(t)\cdot R(t) \approx \Psi(t)\cdot R$$

with *R* = stellar return fraction (simple integral over IMF!!)

➔ Equation for the gas mass

$$\frac{dg}{dt} = -(1-R)\Psi(t) =: -\alpha\Psi(t)$$

IRA (continued)

$$\begin{aligned}\int_{m_{\min}(t)}^{m_{\max}} E_i(m)\Phi(m)\Psi(t-\tau_{\rm MS}(m))dm &\approx \Psi(t)\int_{m_{\min}(t)}^{m_{\max}} p_i(m)\Phi(m)dm \\ &\quad +\Psi(t)\cdot Z(t)\cdot R(t) \\ &=: +\Psi(t)(\alpha y_i - Z(t)\cdot R)\end{aligned}$$

with α = 1-R locked-up mass fraction
y = yield of metal i (another simple integral over IMF!!)

➔ Equation for the metal mass

$$\frac{dgZ_i}{dt} = \alpha\Psi(t)(y_i - Z_i(t))$$

… and for the metallicity

$$g\frac{dZ_i}{dt} = \alpha\Psi(t)\cdot y_i$$

IRA Solution

The mass **s** in long-lived stars and remnants

$$\frac{ds}{dt} = \alpha\Psi(t)$$

For convenience: the gas fraction **f = g/(g+s)**

$$\frac{df}{dt} = \frac{f}{g}\frac{dg}{dt} = -\alpha\Psi(t)\frac{f}{g}$$

➔ Equation for metallicity is **independent of SFR or SF history (SFH)**

$$\frac{dZ}{df} = \frac{dZ/dt}{df/dt} = -\frac{y_i}{f}$$

IRA Solution: The Simple Model

For a constant yield, the metallicity of the gas depends on the **current** gas fraction, **independent of SFR or SF history**

$$Z = - y * \ln f$$

Yield:
= average metal production of the stellar population
- stellar nucleosynthesis
- IMF

Gas fraction:
= current state of galaxy
- galactic gas consumption

But the age-metallicity relation requires explicit knowledge of SFR:

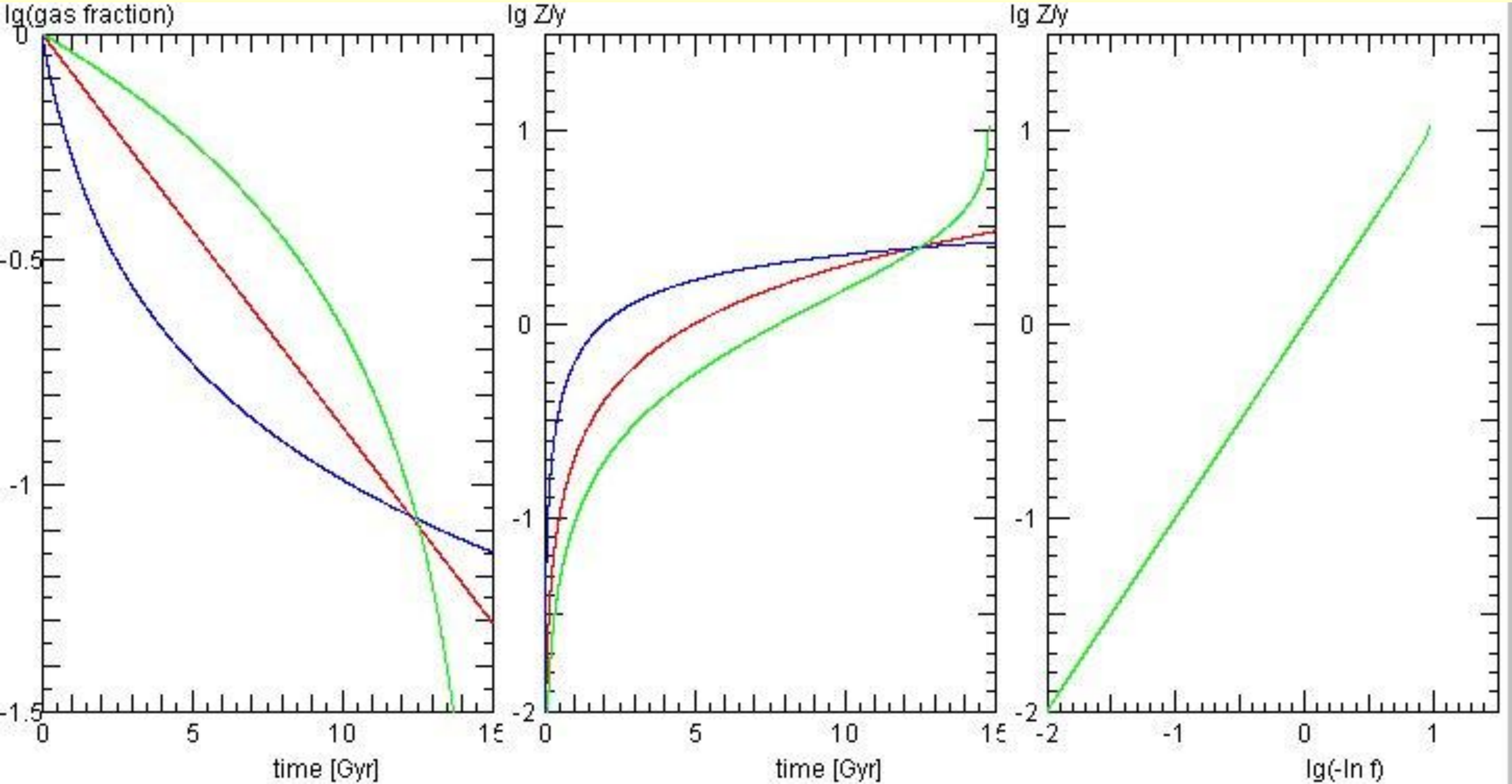
$$g \frac{dZ_i}{dt} = \alpha \Psi(t) \cdot y_i$$

as well as the history of gas consumption:

$$\frac{dg}{dt} = -(1 - R)\Psi(t) =: -\alpha \Psi(t)$$

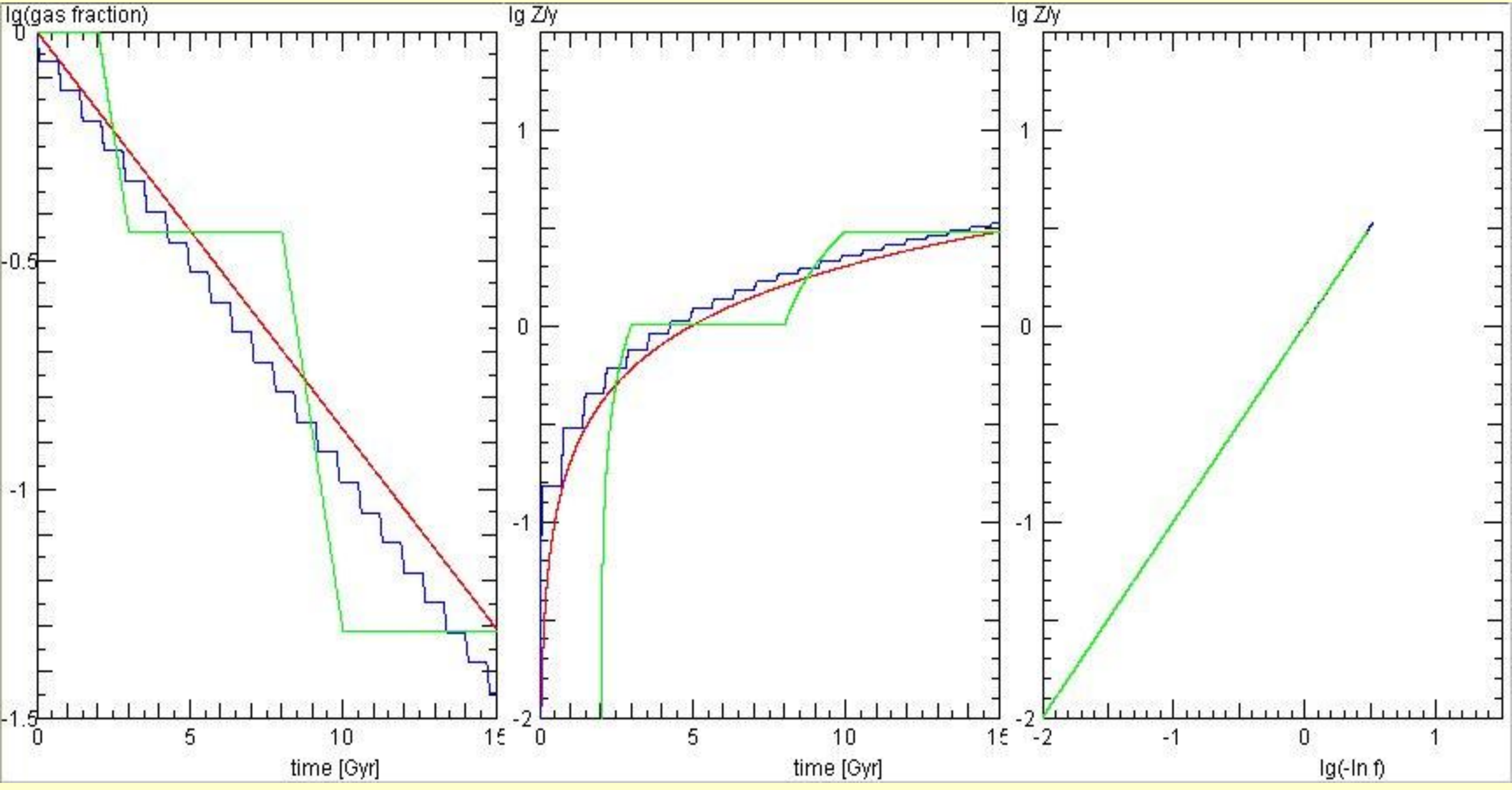
Dependence on SFR = cst. * g^n

n = 1 2 0.25



Dependence on SFH

Short sharp starbursts **Two long starbursts**



Average stellar metallicity:

$$\langle Z \rangle := \frac{1}{s}\int_0^s Z(s')ds'$$

The equation

$$s\frac{d\langle Z\rangle}{dt} = \alpha\Psi(t)\cdot(Z - \langle Z\rangle)$$

has the solution

$$\langle Z\rangle = y\left(1 + \frac{f\ln f}{1-f}\right) \rightarrow y$$

… again, independent of SFR or SFH!

Stellar AbundanceDistributionFunction

- long-lived stars = mixture of old and young stars … and we don't need to determine individual ages … WOW!!!
- Histogram of their metallicities (in IRA):

$$\frac{ds}{dZ} = \frac{ds/dt}{dZ/dt} = \frac{\alpha\Psi}{y\alpha\Psi/g} = \frac{g}{y}$$

with

$$\frac{dg}{dZ} = \frac{-\alpha\Psi}{y\alpha\Psi/g} = -\frac{g}{y}$$

➔ **Simple Model:** $$\frac{ds}{dZ} = \frac{1}{y}\exp(-Z/y)$$

… once again, independent of SFR or SFH!

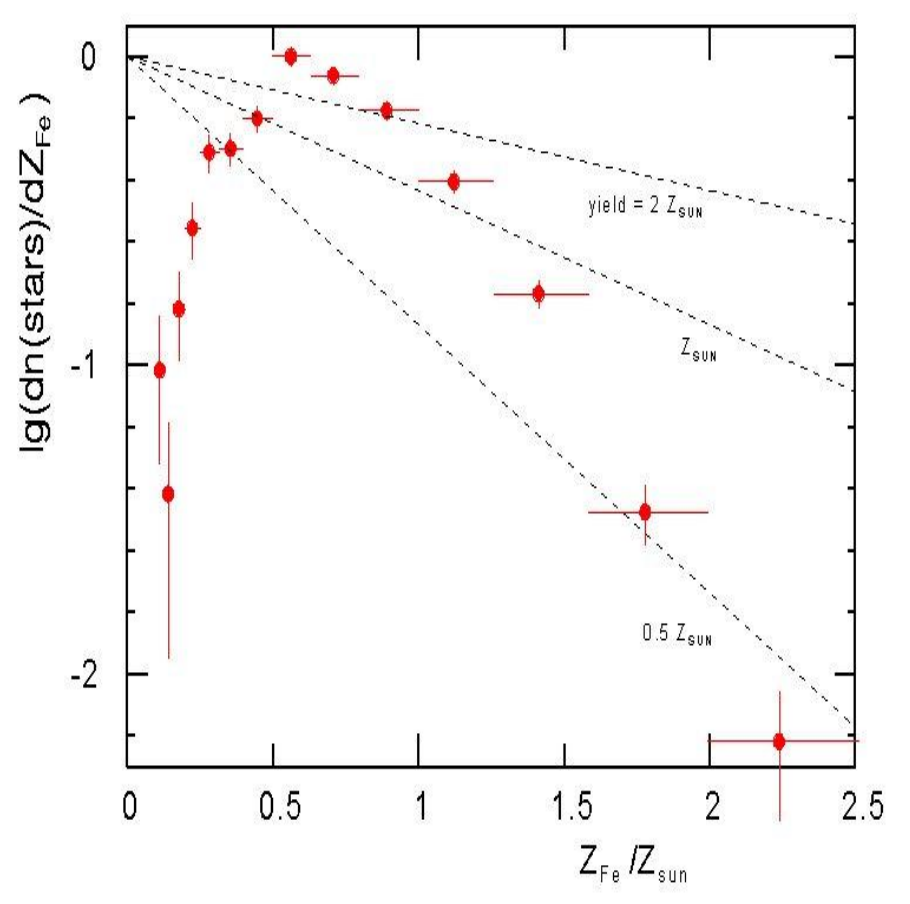
Two Forms of the AbundanceDistributionFunction

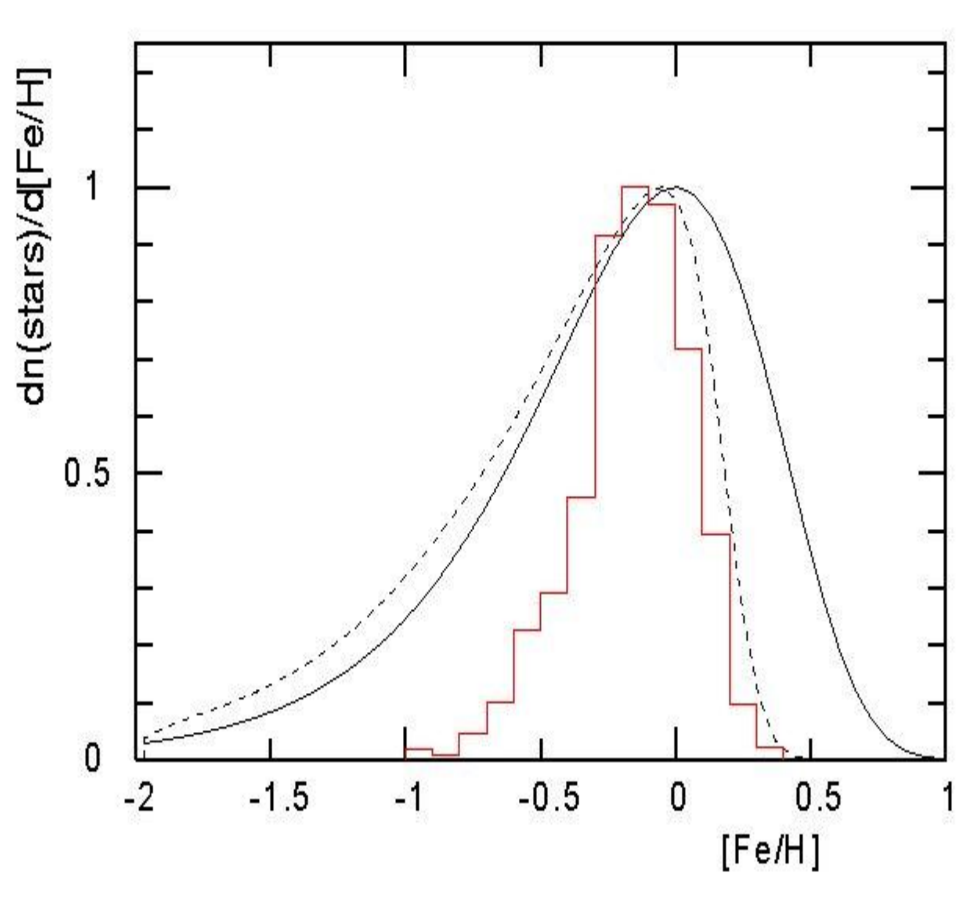
theoretician's pet:

$\log ds/dZ = \text{const.} - Z/y$

observer's preference:

$ds/d\log Z = Z/y \exp(-Z/y)$





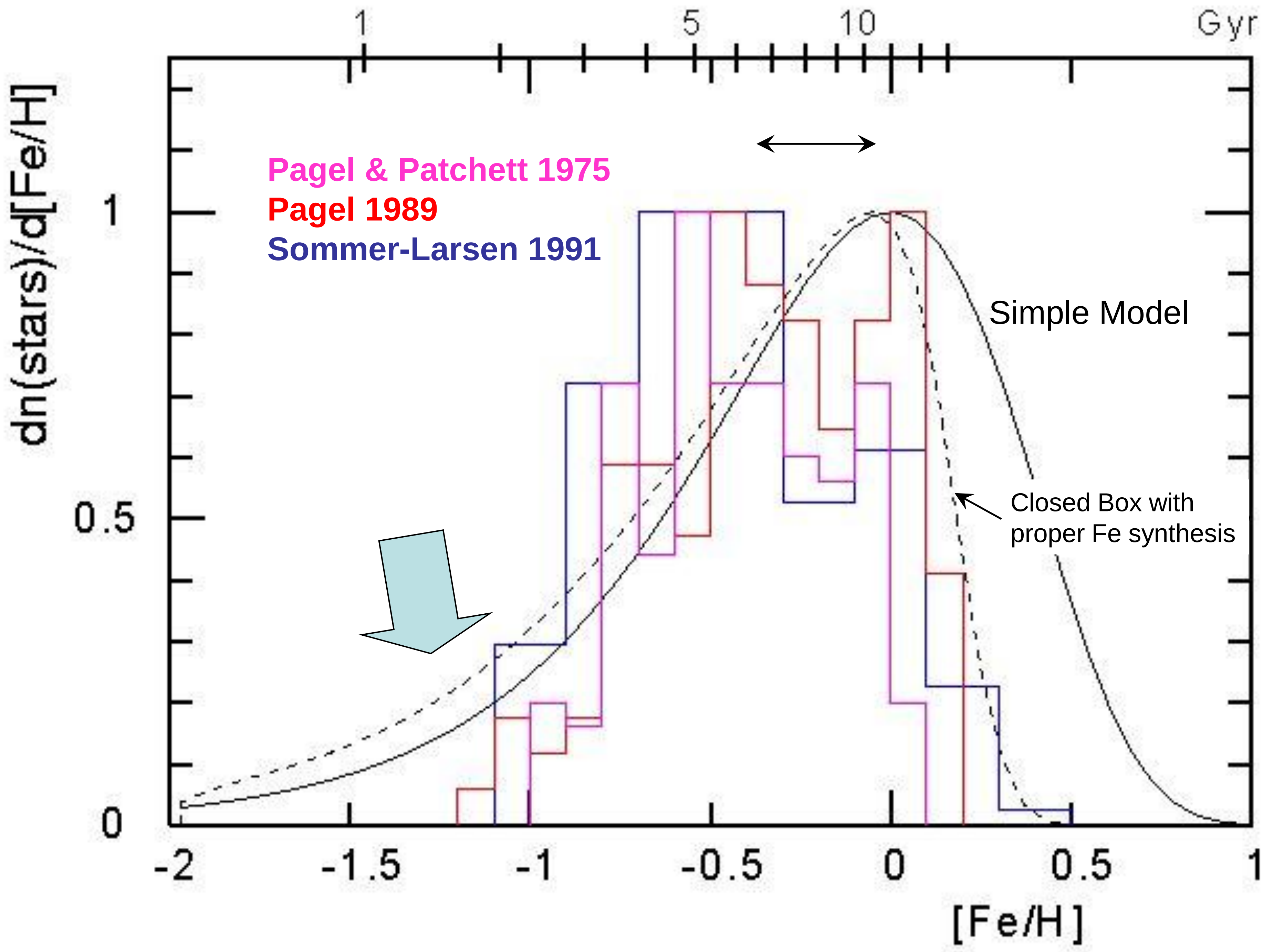
(- - - with proper Fe synthesis)

G dwarf problem: history I

- Van den Bergh (1962), Schmidt (1963): solar nh'd has paucity of metal-poor G-dwarfs compared to Simple Model
- ➔ alternative models …

- Pagel & Patchett (1975): 132 stars, photometric [Fe/H]
- … chemical evolution modelers often use infall/inflow of metal-poor gas into solar nh'd

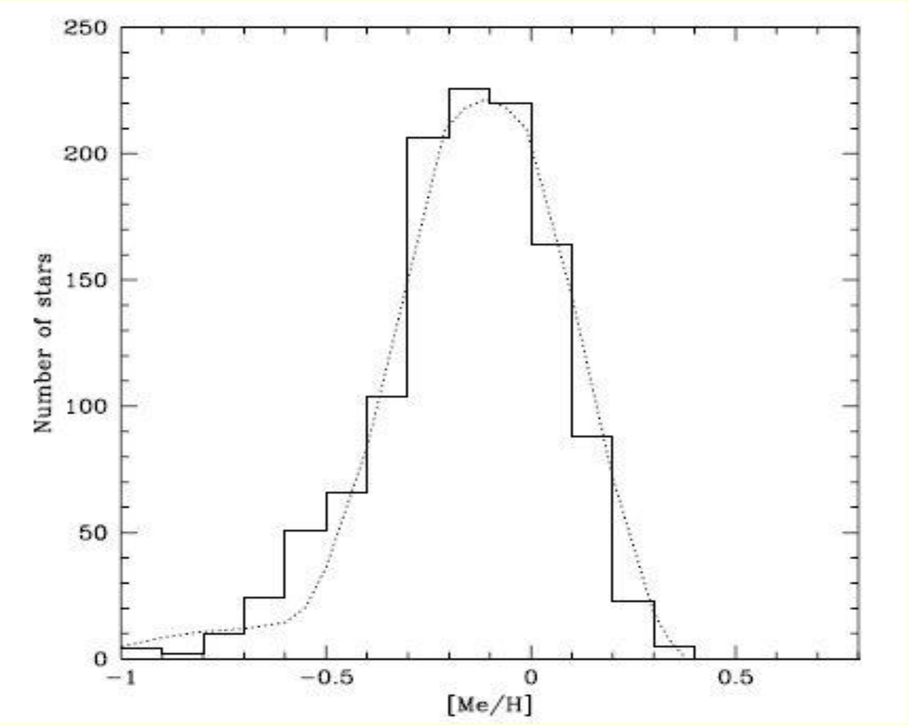
- Pagel (1989): improved UV excess - [Fe/H] relation
- Sommer-Larsen (1991): correction for different heights above plane (old stars have higher velocity dispersion ➔ greater scale heights): discrepancy still there!



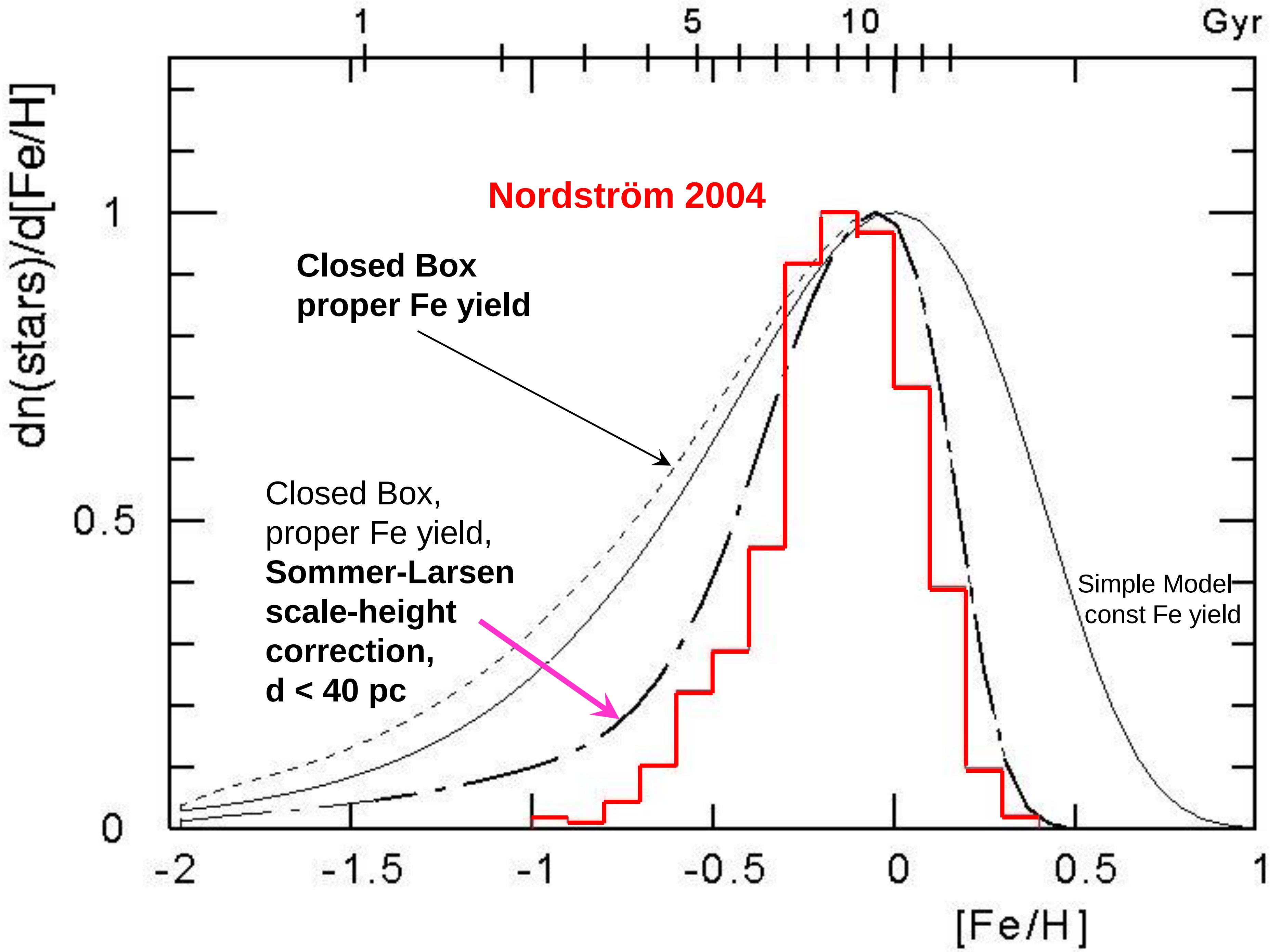
Gyr
1
5
10
Pagel & Patchett 1975
Pagel 1989
Sommer-Larsen 1991
Simple Model
Closed Box with
proper Fe synthesis
dn(stars)/d[Fe/H]
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
[Fe/H]

G dwarf problem: history II

- Several new data sets …
- … Nordström et al. (2004): 1195 stars with spectroscopic/photometric [Fe/H]: worse than ever!



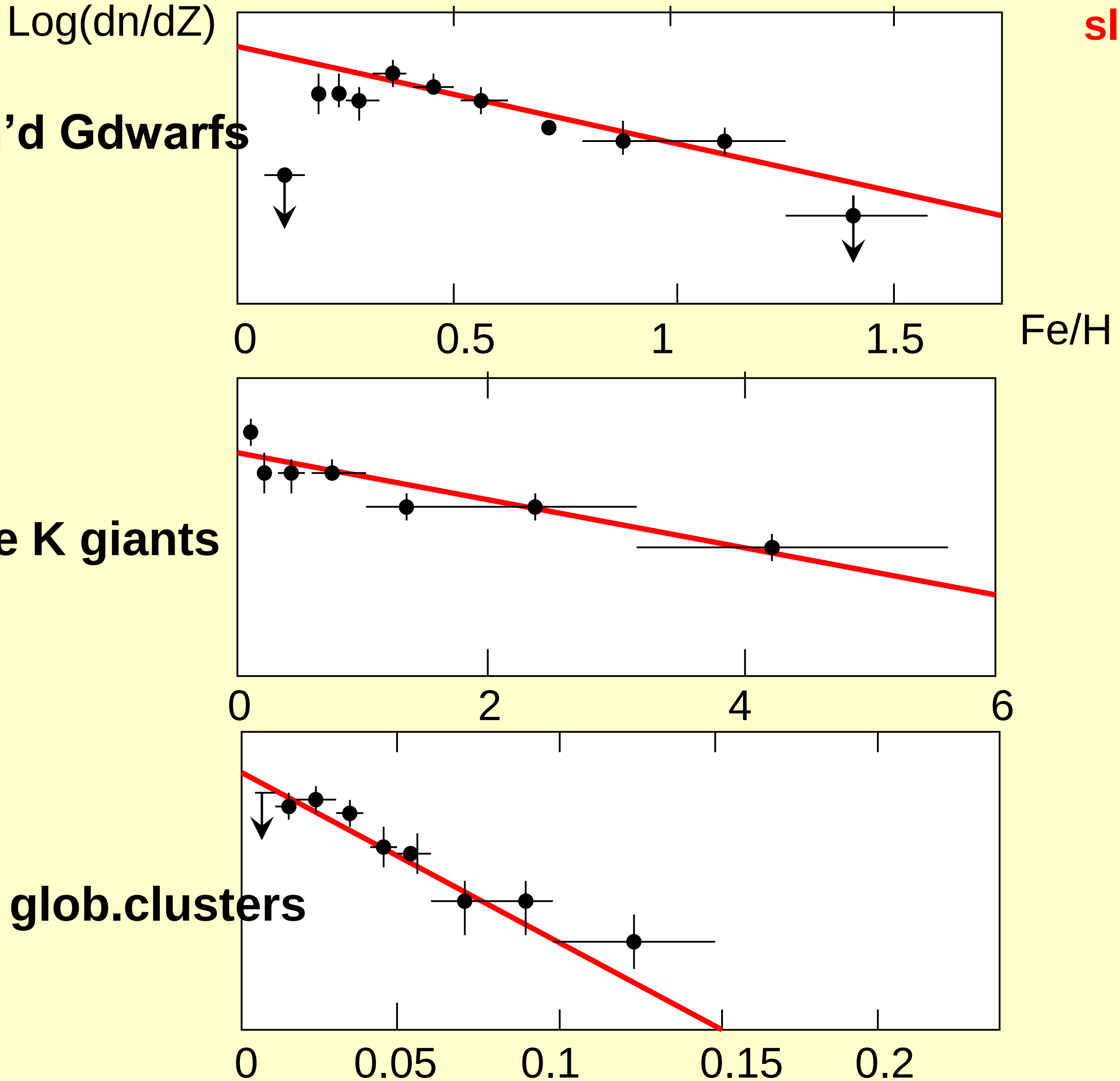
- Haywood (2006): Closed Box model with scale height correction does the job!



1
5
10
Gyr
Nordström 2004
Closed Box
proper Fe yield
Closed Box,
proper Fe yield,
Sommer-Larsen
scale-height
correction,
d < 40 pc
Simple Model
const Fe yield
dn(stars)/d[Fe/H]
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
[Fe/H]

Stellar ADFs in

slope ➔ eff.yield



Sol.Nh'd Gdwarfs = Disk

0.4 $Z_\odot$

Bulge K giants

1.8 $Z_\odot$

Halo glob.clusters

0.025 $Z_\odot$

after Pagel 1989

Why are the yields different?

- Different nucleosyntheses in Bulge/Disk/Halo stars?
- Different IMFs?
- Different evolutions/conditions/processes?
- …
- ??

Abundance ratios …

If the yield of element *i* depends on metallicity of an element with **constant yield (‘primary production’)**:

$$y_i = y_{i0} Z^k$$

k = 0 primary
k = 1 secondary
k = 2 tertiary …

in IRA one gets this equation

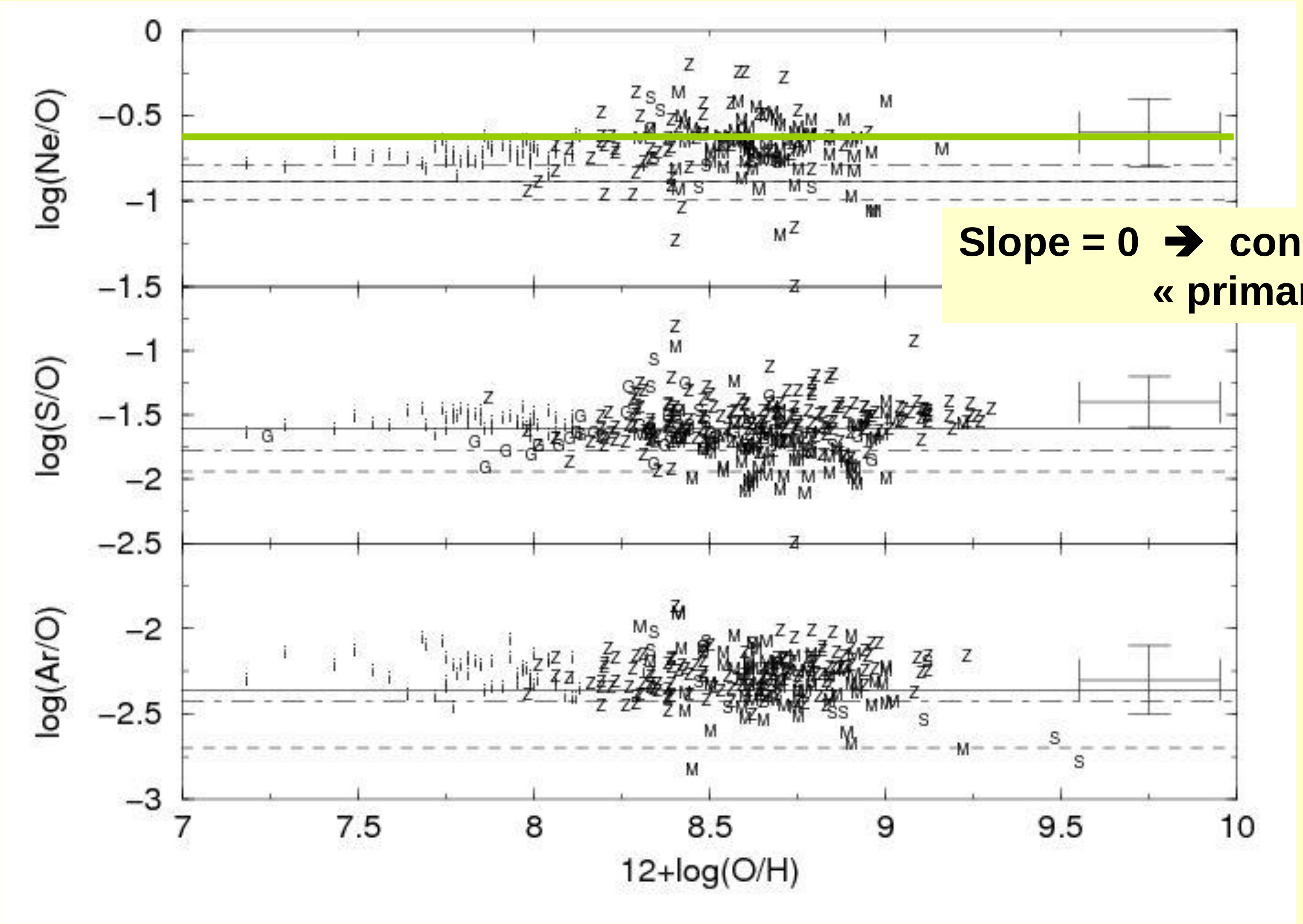
$$\frac{dZ_i}{dZ} = \frac{y_{i0}}{y_Z} Z^k$$

and the solution

$$\frac{Z_i}{Z} = \frac{y_{i0}}{(k+1) y_Z} Z^k$$

➔The exponent ***k*** (= slope in a log-log plot) of the abundance ratio as a function of metallicity indicates the metallicity-dependence of the yield, and thus tells about the nucleosynthesis of the element *i*

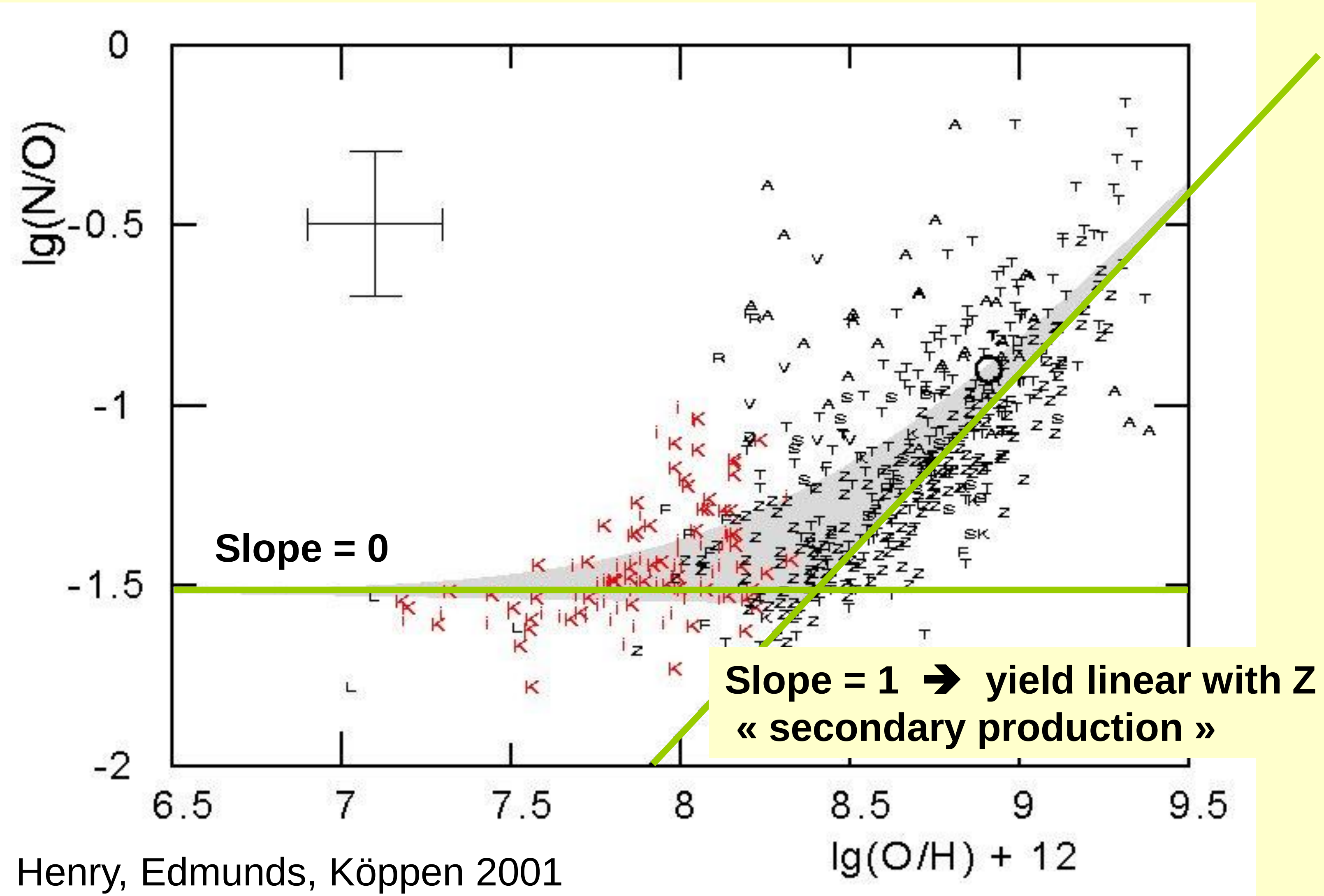
Abundance ratios: O,Ne,S,Ar



Slope = 0 ➔ constant yield
« primary production »

Henry & Worthey 2000

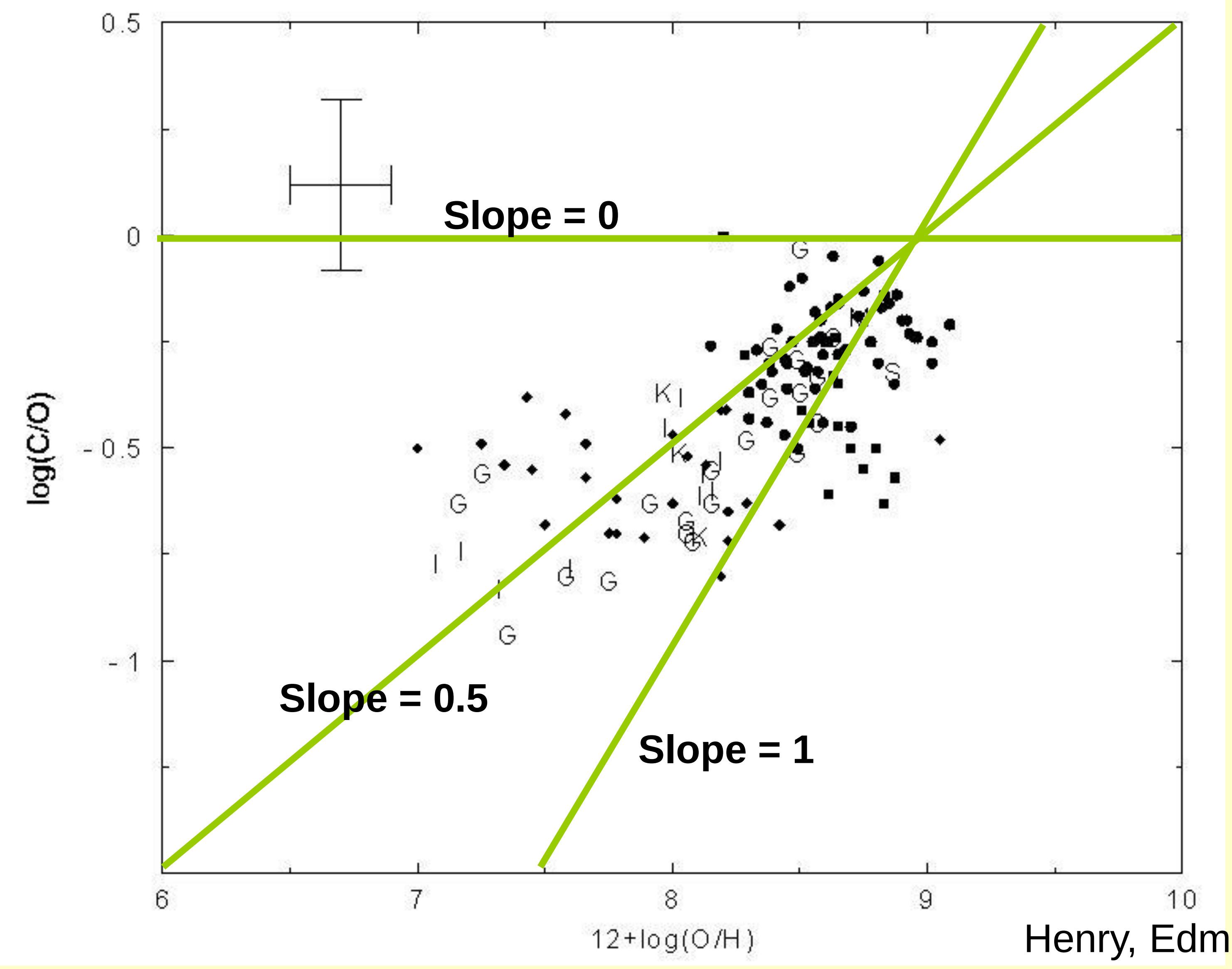
Abundance ratios: N/O



???

Henry, Edmunds, Köppen 2001

Abundance ratios: C/O



???

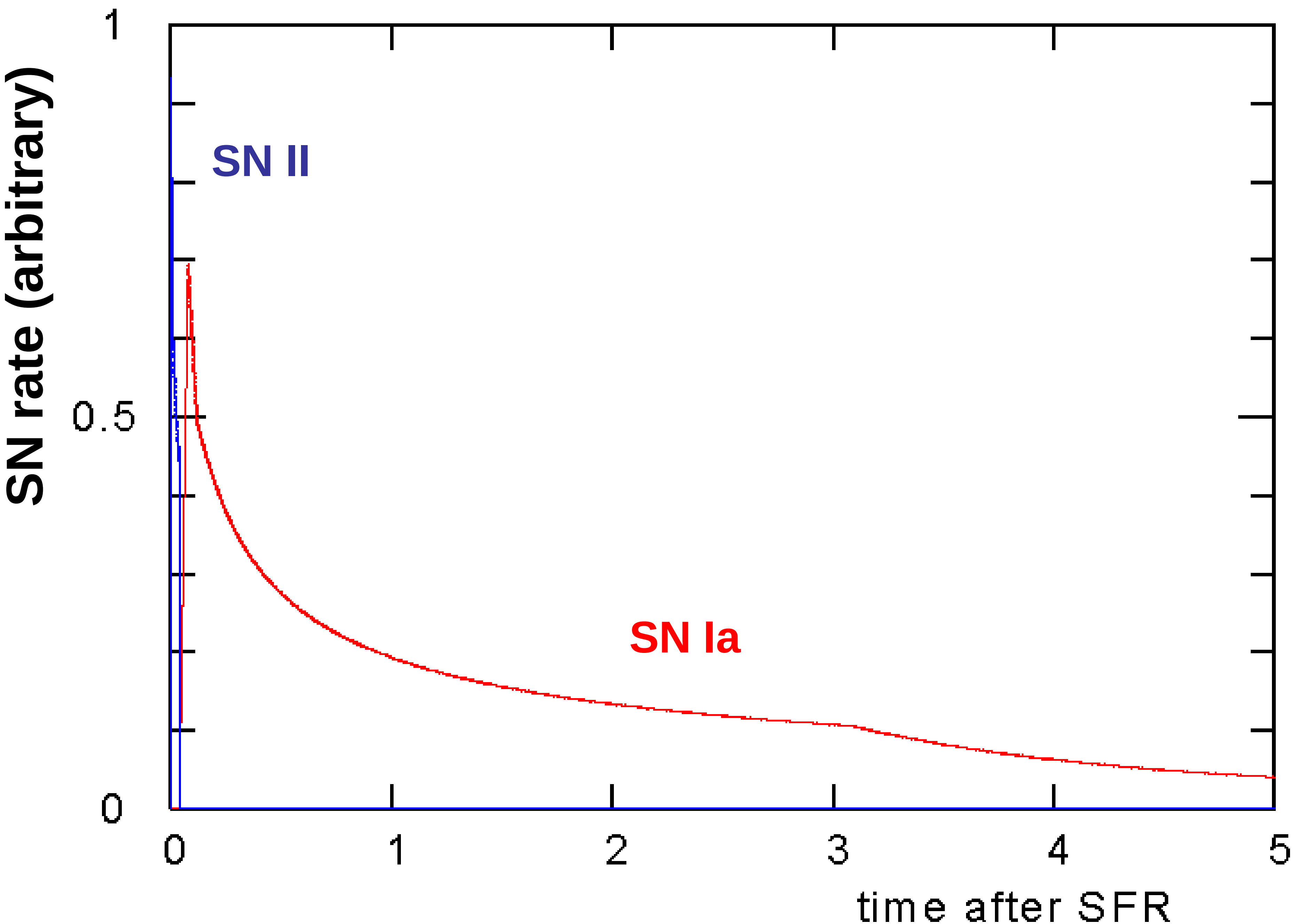
Henry, Edmunds, Köppen 2001

C-N-O is a longer story …(see later)

Fe

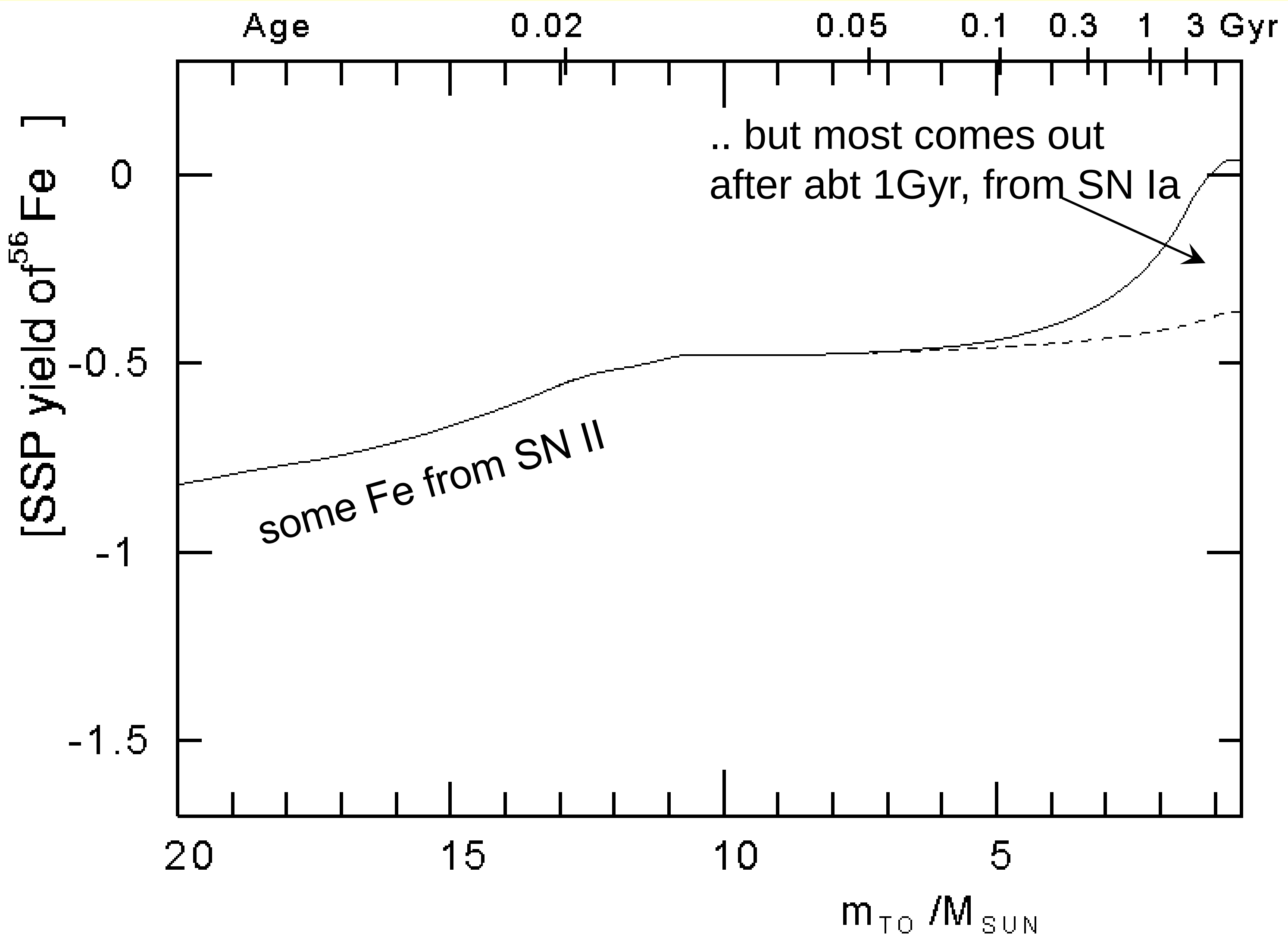
- Some comes from core-collapse SN (‘type II SN’): about < 0.15 $M_{\odot}$ … depends on the mass of the remnant
- From C-deflagration SN (‘type Ia SN’): about 0.6 $M_{\odot}$ per event (Model 7 of Nomoto et al.)

- Need fraction of stars that evolve as binaries: about 0.035 (but free parameter)

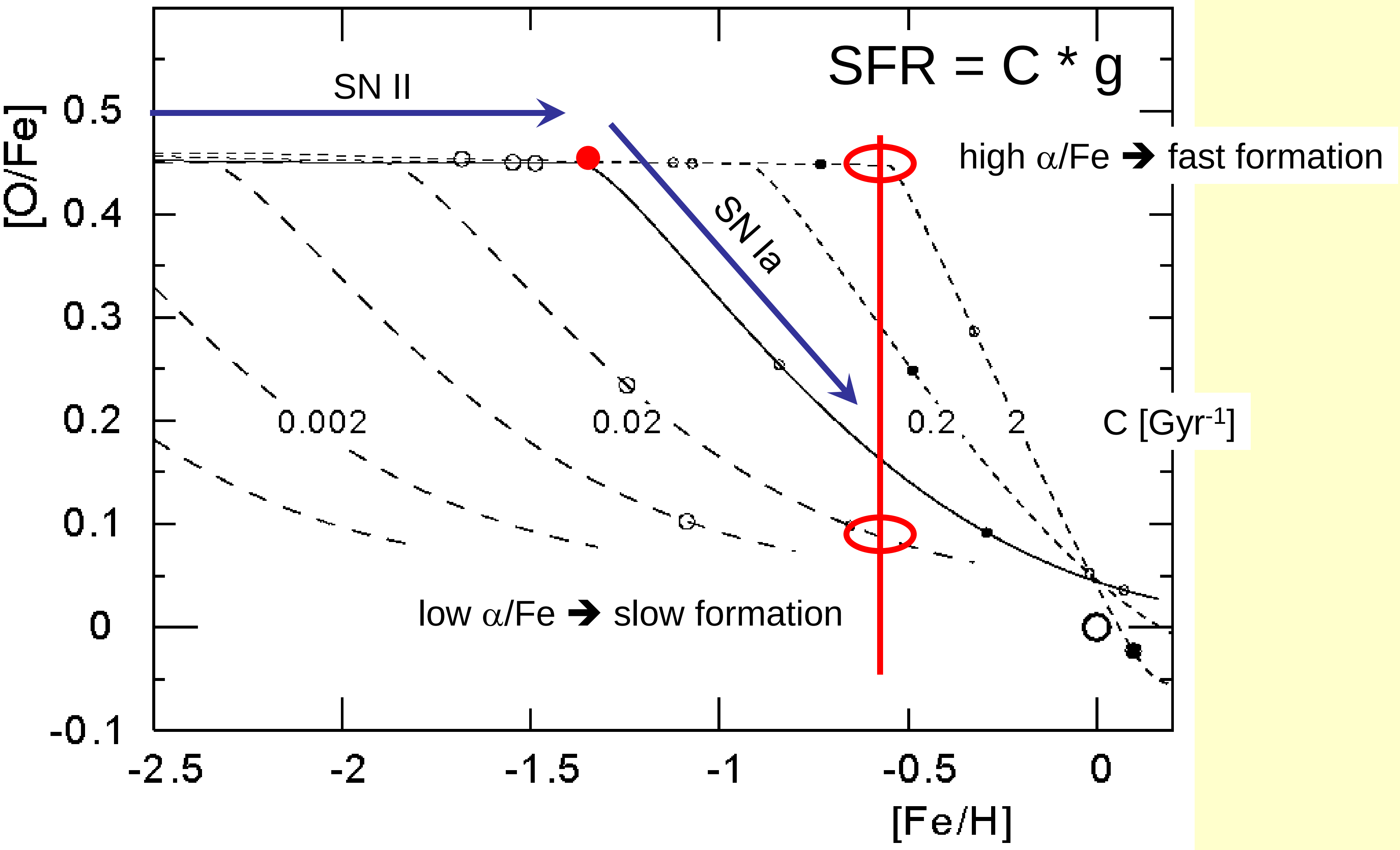


SN rate (arbitrary)
1
0.5
0
SN II
SN Ia
0
1
2
3
4
5
time after SFR

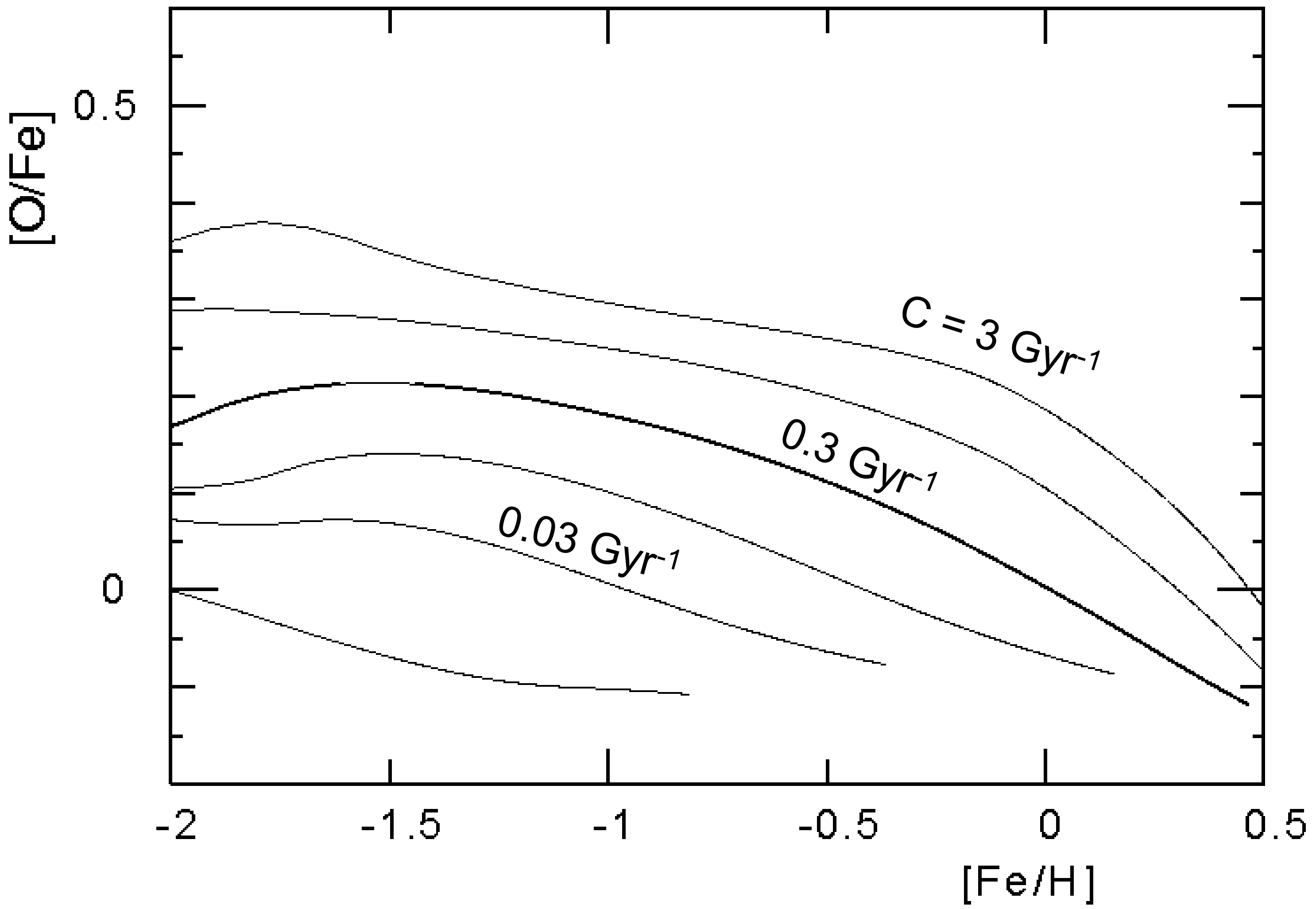
SSP yield for iron



O/Fe (or α/Fe) ratio measures SFR time-scale (illustrative model)

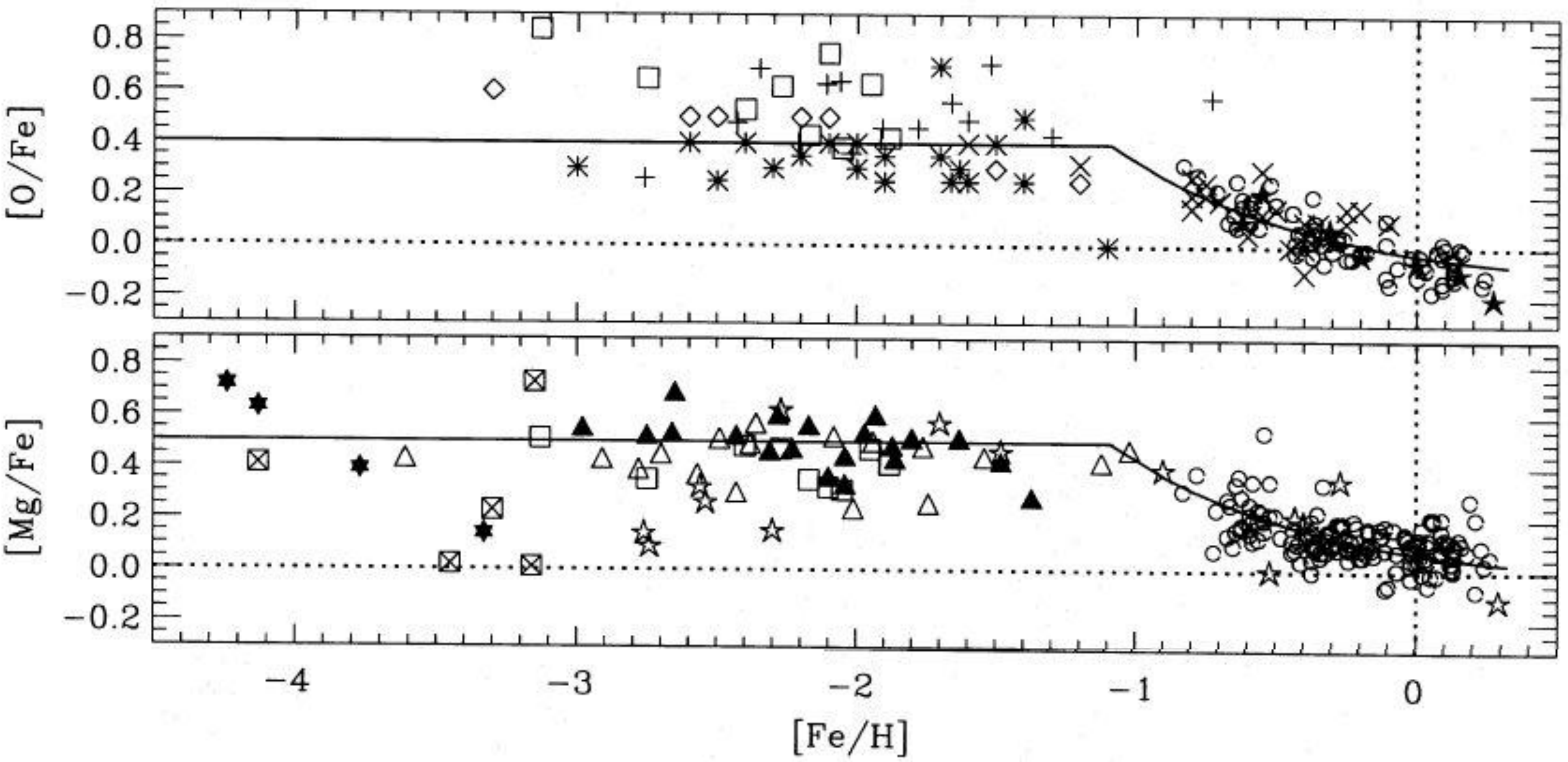


O/Fe (or α/Fe) ratio measures SFR time-scale (real nucleosynth. data)



Nucleo:
HG, WW95
IMF: Salpeter
SFR = C*g

Abundance ratios: O/Fe, Mg/Fe, …



Pagel & Tautvaišienė 1995

Properties of the Simple Model

Independent of SFR and SFH:

primary metallicity (y_p=const.)	$Z_p = - y_p \ln f$
Z-depend. metallicity	$y \propto Z^n_p \rightarrow Z/Z_p \propto Z^n_p$
stellar ADF	log dn/dZ = -Z/y + const.
peak of ADF (dn/dlogZ)	$Z^{peak} = y_p$
mean stellar metallicity:	$\langle Z \rangle = y(1 + \frac{f \ln f}{1-f}) < y$

Beyond the Simple Model

The G-dwarf problem lead to think about

Deviations from the Simple Model:

- The initial metallicity was NOT zero: **P**rompt **I**nitial **E**nrichment, Hypernovae
- The box is NOT closed: gas inflow/outflow
- The yield is NOT constant
- Metallicity-enhanced star formation
- …
- and all these ideas did give better fits …

Deviations from the Simple Model:

- Gas outflows
- (Enriched outflows = loss of stellar ejecta)
- Gas accretion: inflows/infall
- (Gas throughflow)
- Imperfect mixing of the gas
- Multiple components
- Sudden events

Outflows of interstellar gas

Arbitrary outflow rate: W(t) ➔ general function w(t) = W/$\alpha\Psi$ $\geqslant$0

$$\frac{dg}{dt} = -\alpha\Psi(t) - W(t) =: -\alpha\Psi(t)(1 + w(t))$$

$$\frac{dgZ_i}{dt} = \alpha\Psi(t)(y_i - Z(t)(1 + w(t)))$$

➔

$$g\frac{dZ_i}{dt} = \alpha\Psi(t) \cdot y_i$$

Same as Simple Model!

Solution with constant w > 0

$$Z_i = -\frac{y_i}{1 + w} \ln f$$

= Simple Model with reduced yield!

and the G-dwarfs:

$$\frac{ds}{dZ} = \frac{1}{y} \exp(-\frac{Z}{y/(1 + w)})$$

= Simple Model with reduced yield!

It can be shown (by num.eperiments) and also proven that there is no function w(t) that would explain the observed G-dwarf distribution

Enriched outflows

- SN explosion carries out of the galaxy part or all of the freshly produced metals (‘blow-out’, ‘blow-away’)
- reduces the yield by some factor averaged over the stellar population …
- factor depends on the gravitational field of the galaxy, the position where the SN explodes (requires detailed hydrodynamical calculation of the SN explosion and its effect on the ISM)

Enriched outflows

- one could also introduce a certain time evolution of the reduction factor of the yield … and play with that …

Accretion of (primordial) gas

Arbitrary accretion rate: A(t) ➔ accretion factor $a(t) = A/\alpha\Psi \gtrless 0$

$$\begin{aligned} \dot{g} &= -\alpha\Psi + A \\ \dot{s} &= \alpha\Psi \\ \dot{z} &= -Z\alpha\Psi + p\alpha\Psi \end{aligned} \longrightarrow \begin{aligned} \frac{dg}{ds} &= -1 + a \\ g\frac{dZ}{ds} &= p - Za \end{aligned}$$

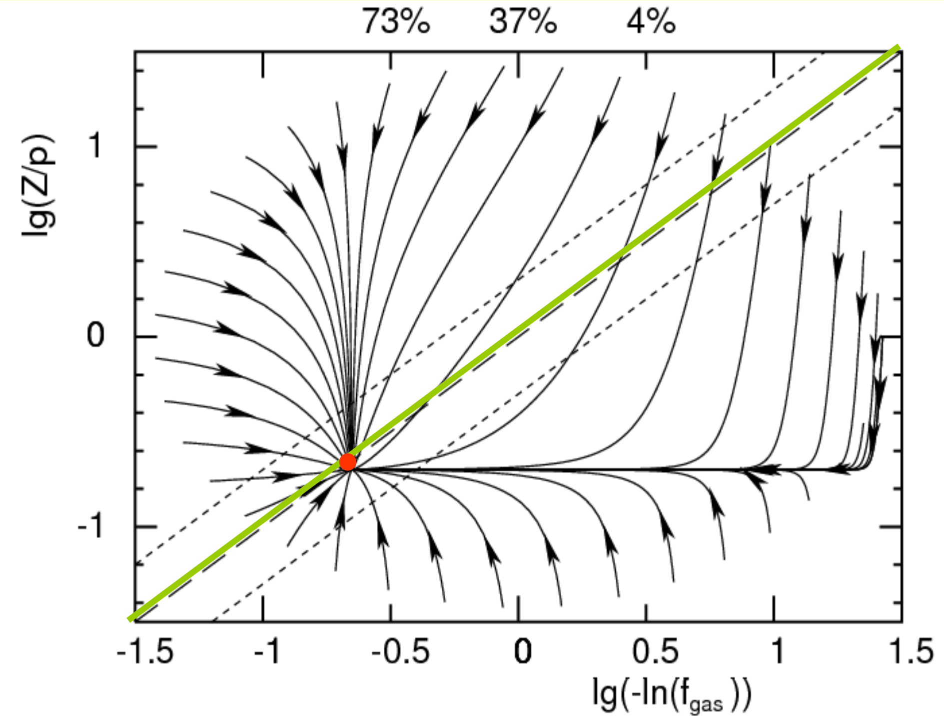
Linear Accretion Models (a=const.) give the clue:
consider the **streamlines** in the f-Z diagram

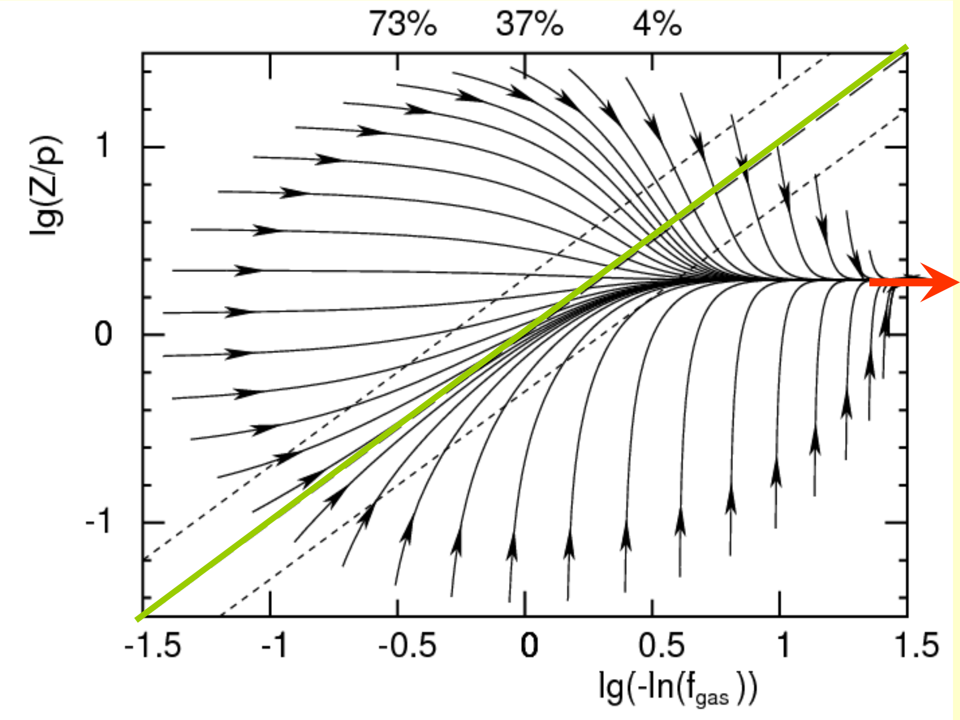
$$\frac{dZ}{d(-\ln f)} = \frac{dZ/ds}{d(-\ln f)/ds} = \frac{p - Za}{1 - a(1-f)}$$

critical points: $f = 1 - 1/a$ (as long as $0 < f < 1$)
$Z = p/a = y/a$

yield: $p = y$ Köppen & Edmunds 1999

Accretion: 2 equilibrium modes

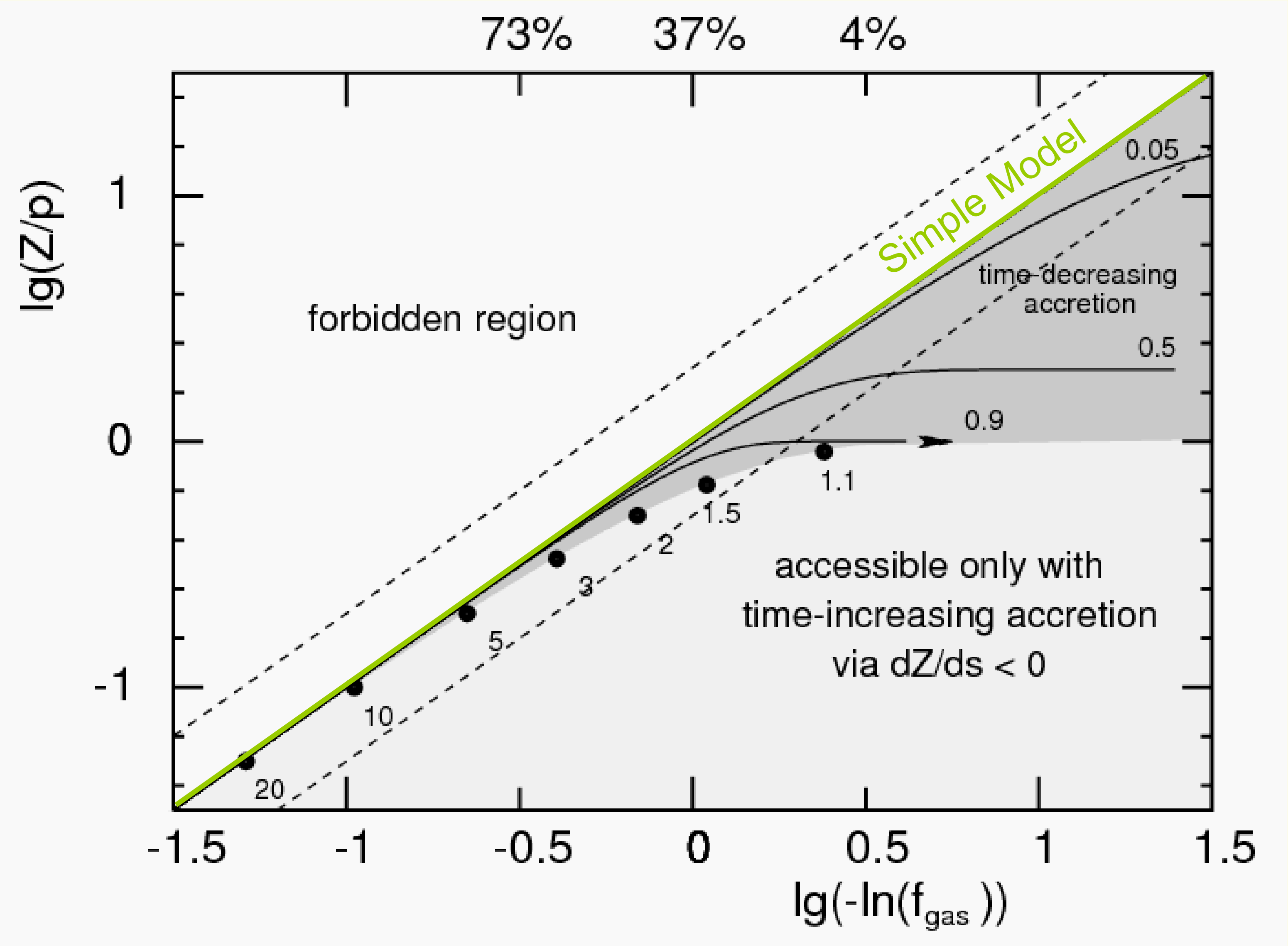




a > 1: both critical conditions are met:
an attracting node =
- gas consumption = gas accretion
- metal production = metal 'dilution'
- but close to **Simple Model**

a < 1: only condition on Z is met:
an attracting funnel (=partial equilibrium):
- continuous gas consumption
- metal production = metal 'dilution'

Behaviour of arbitrary accretion



Accretion: secondary elements

$$R := Z_S/Z$$

$$g\frac{dZ}{ds} = p - aZ$$

$$g\frac{dZ_S}{ds} = p_s Z - aZ_S$$

$$gZ\frac{dR}{ds} = p_s Z - pR$$

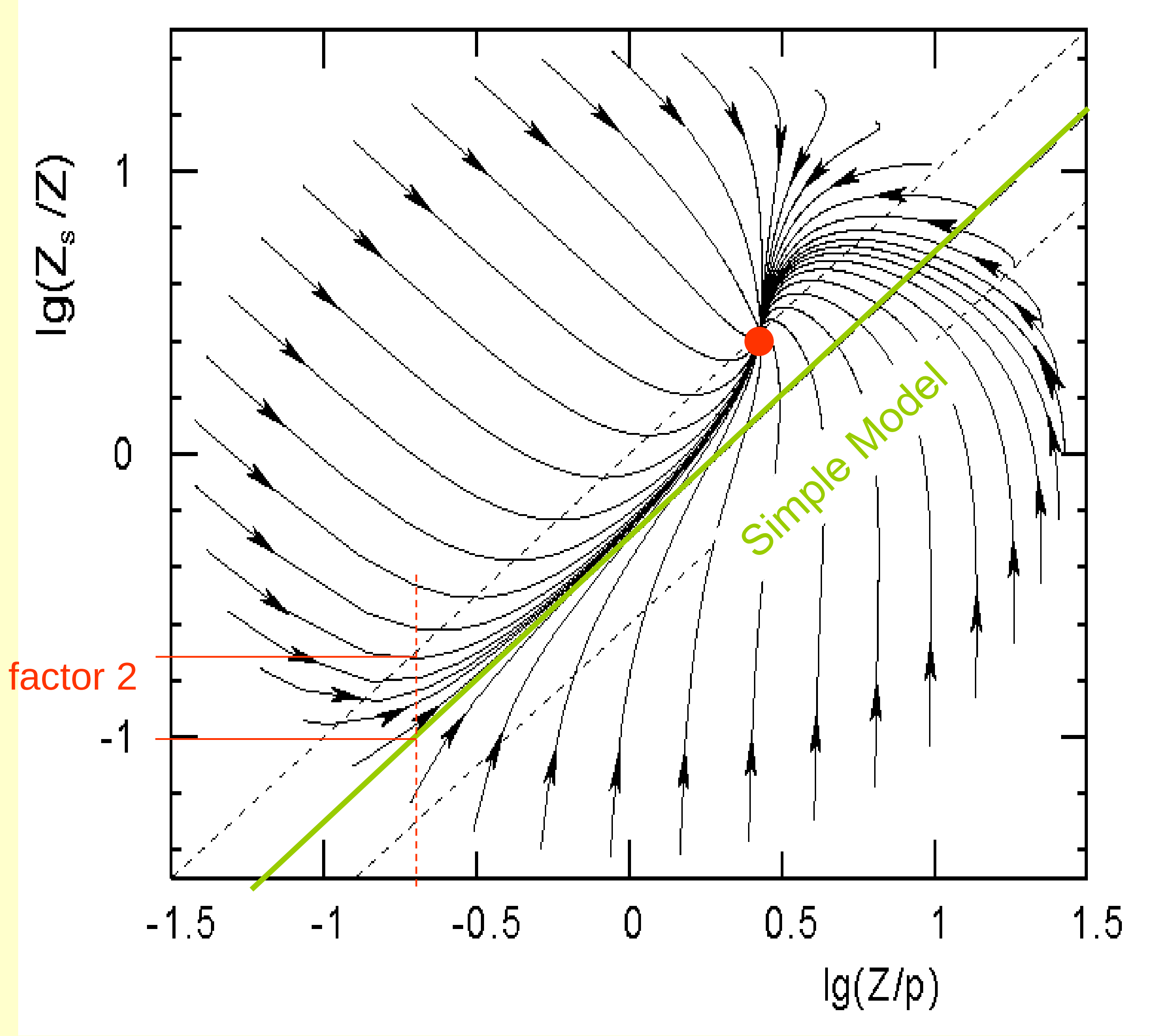
Streamlines:

$$\frac{d\lg R}{d\lg Z} = \frac{1}{R}\frac{p_s Z - pR}{p - aZ}$$

Equilibrium = attr.Node

$Z_S/Z = R = Z * y_S/y$

close to Simple Model

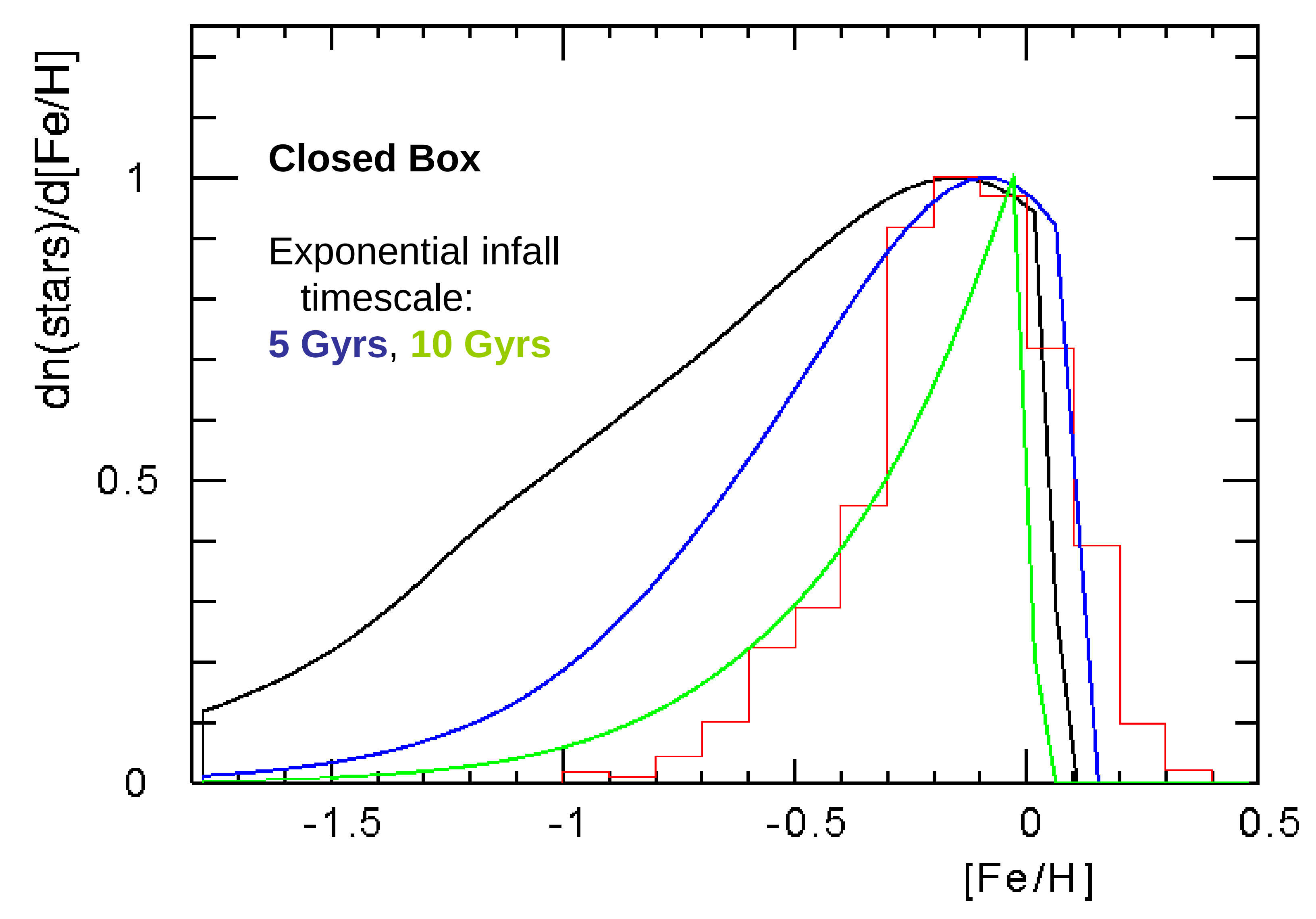


Accretion: summary

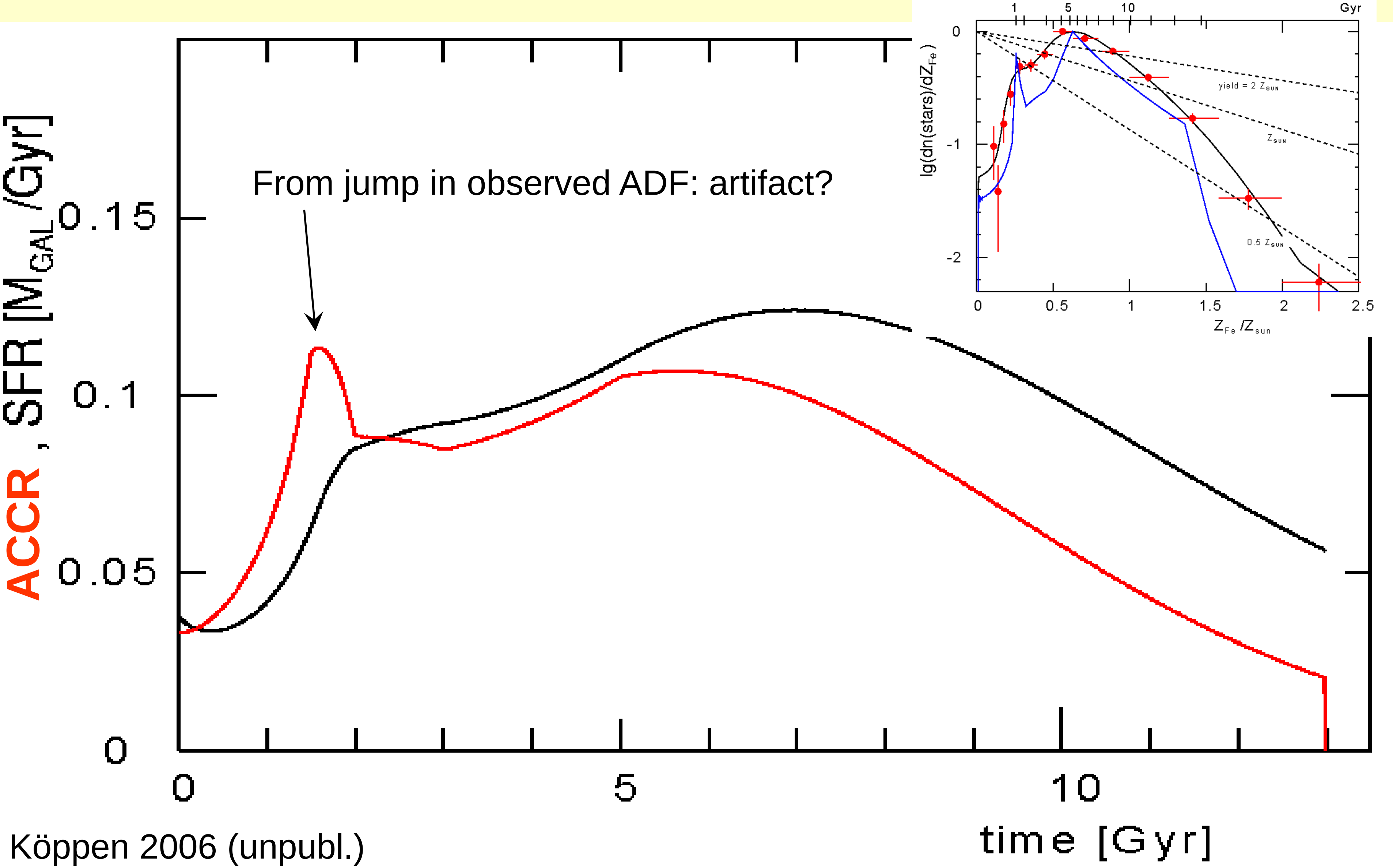
- Accretion of metal-poor gas always reduces the effective yield
- Models with monotonically decreasing a(t) evolve close to the Simple Model
- Rapid and massive accretion onto a gas-poor system can give strong deviations (↓)

- Accretion of metal-rich gas (through-flow models) can push up the yield a bit (but you have to make the metals elsewhere…)

Accretion helps G dwarfs:



One could also deduce infall history



Köppen 2006 (unpubl.)

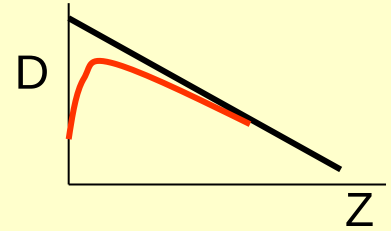
Possibilities to alter the ADF?

$$D(Z) := \frac{ds}{dZ}$$

Take general chemical model (but in IRA):

- metallicity-dependent yield $y(Z)$
- accretion of gas $a(t)$ = acc.rate/SFR
- metallicity of accreted gas $Z_a(t)$
- outflow of gas $w(t)$ = flow.rate/SFR
- escape of stellar ejecta $\eta(t)$ = fraction that escapes

To solve G dwarf problem, need **dD/dZ > 0**



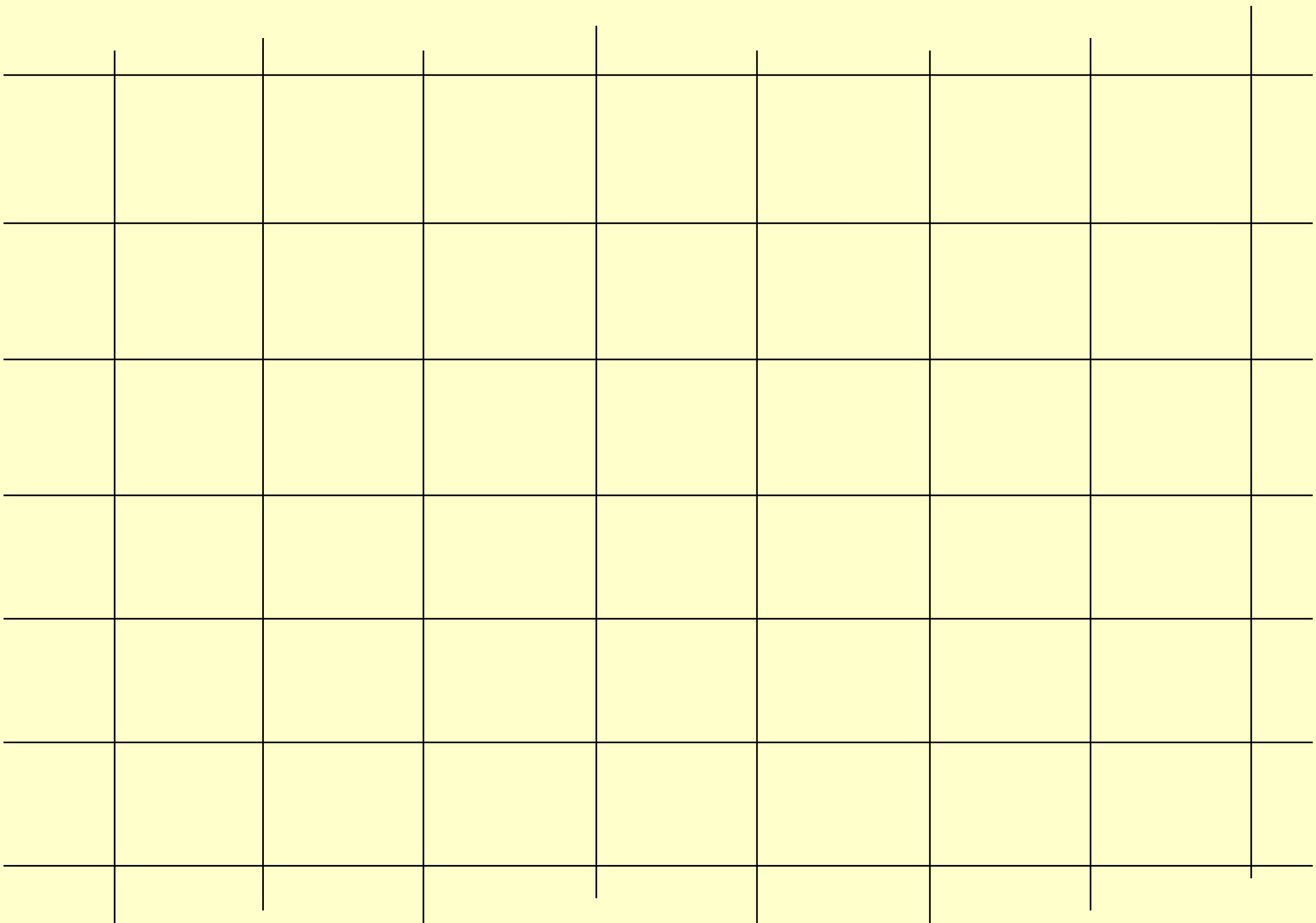
$$\frac{d\ln D}{dZ} = \frac{1}{(1-\eta)y + (Z_a - Z)a}\Big[-1 - (1-\eta)\frac{dy}{dZ} + \eta\Big(1-\frac{1}{\alpha}\Big) + y\frac{d\eta}{dZ} + a\Big(2 - \frac{dZ_a}{dZ}\Big) - (Z_a - Z)\frac{da}{dZ} - w\Big]$$

Low yield or strong metal loss at high Z

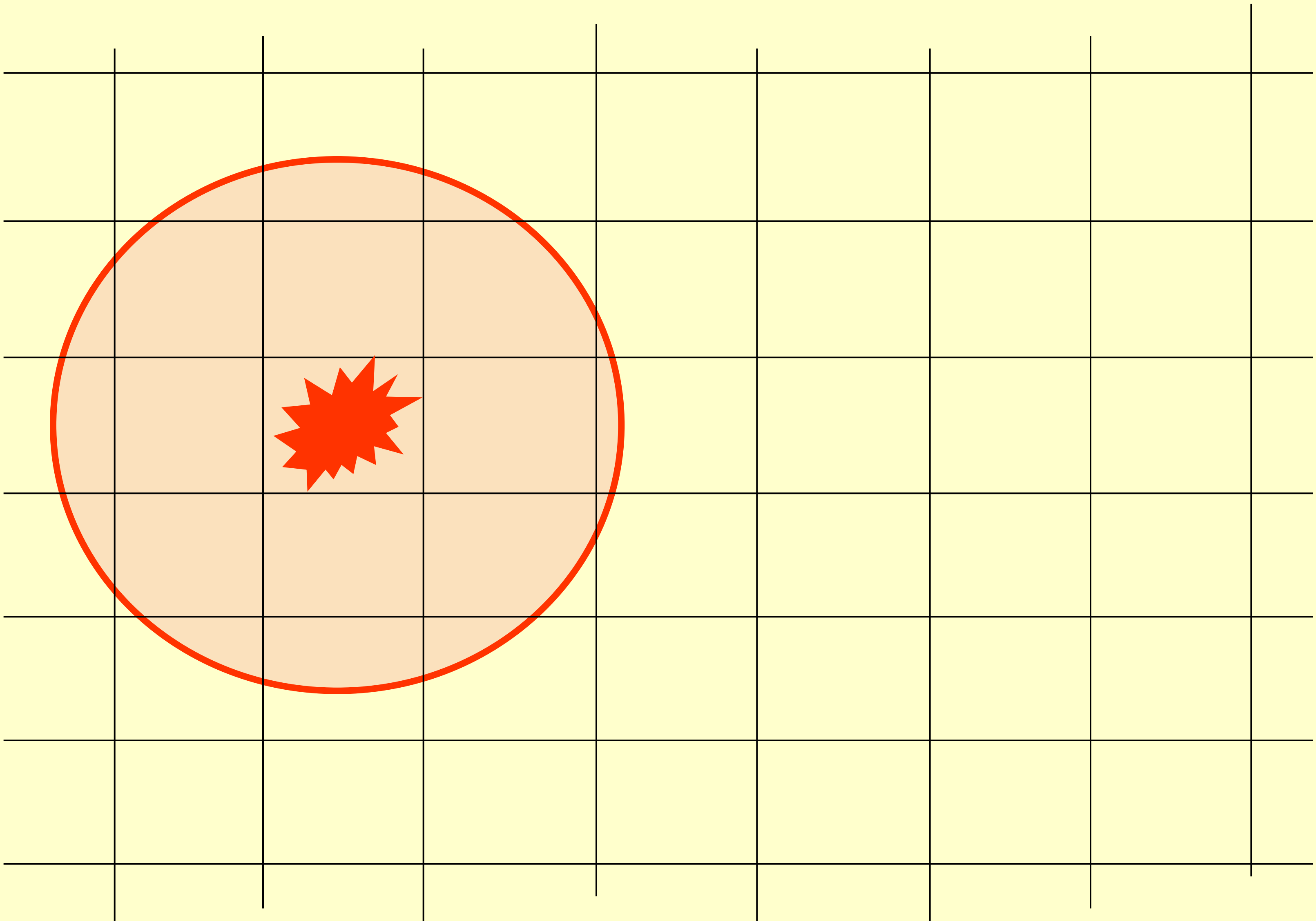
Accretion (Infall, Inflow) ☺

Outflows? NO! ☹

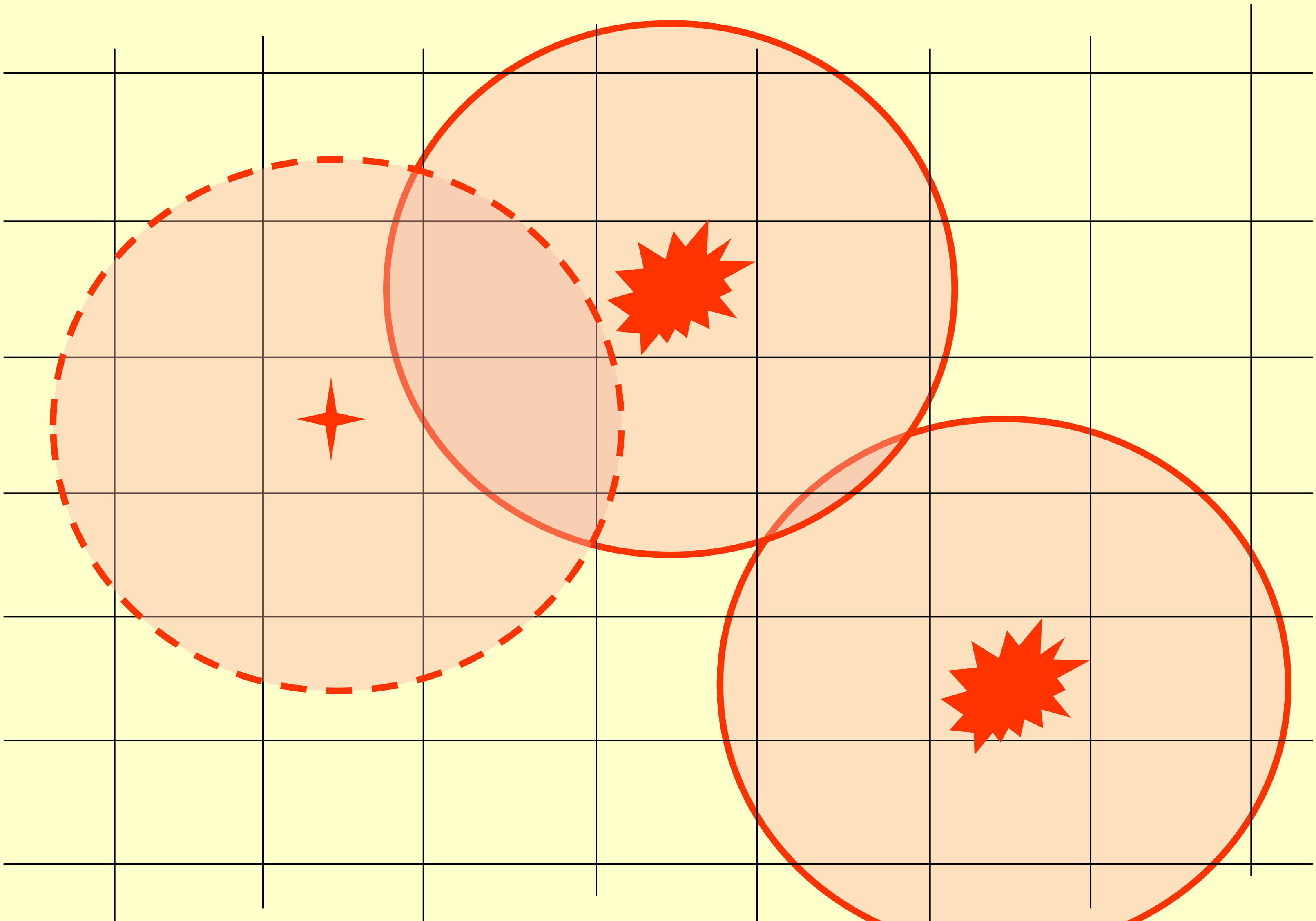
Imperfect mixing …



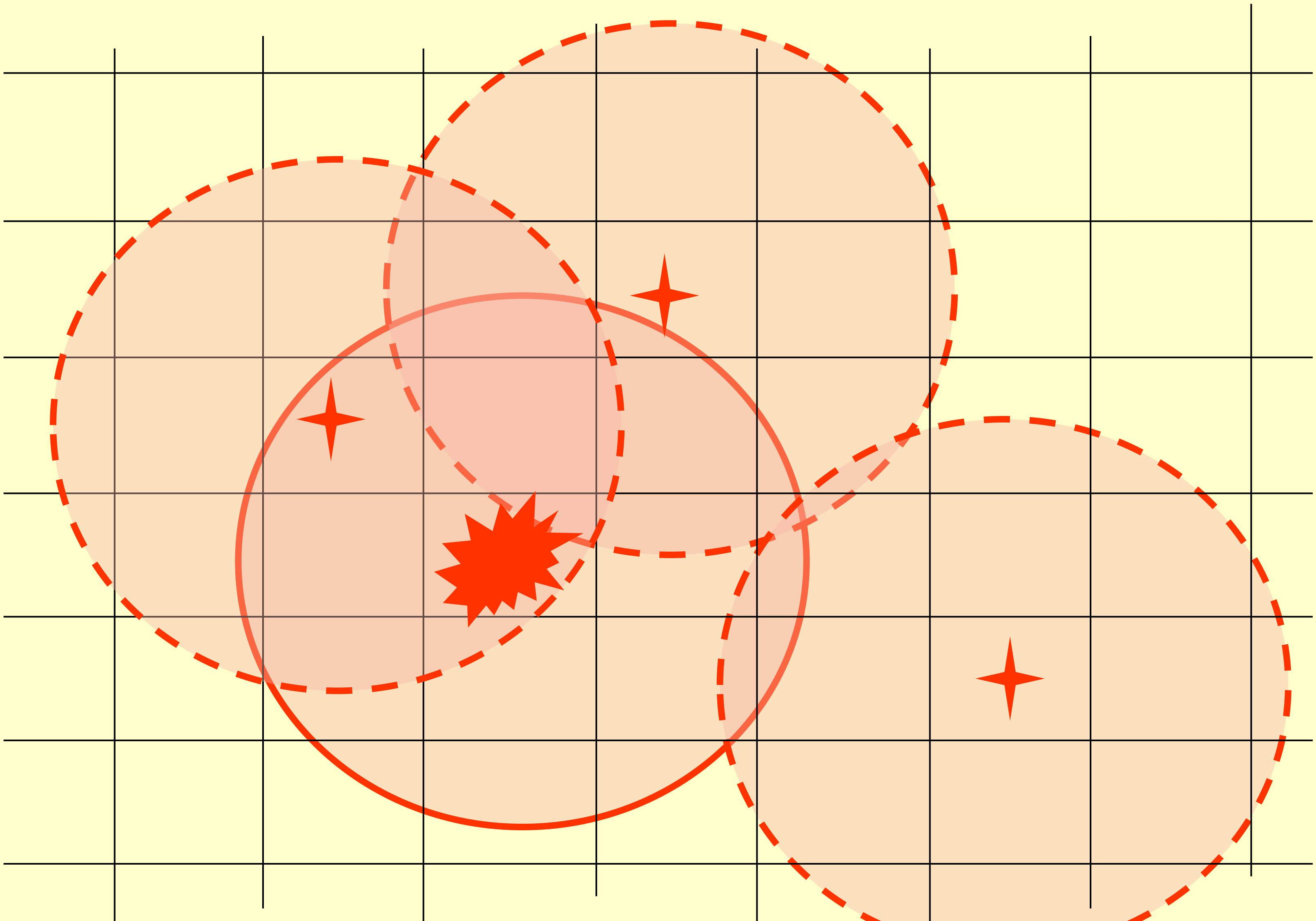
after 1st time step



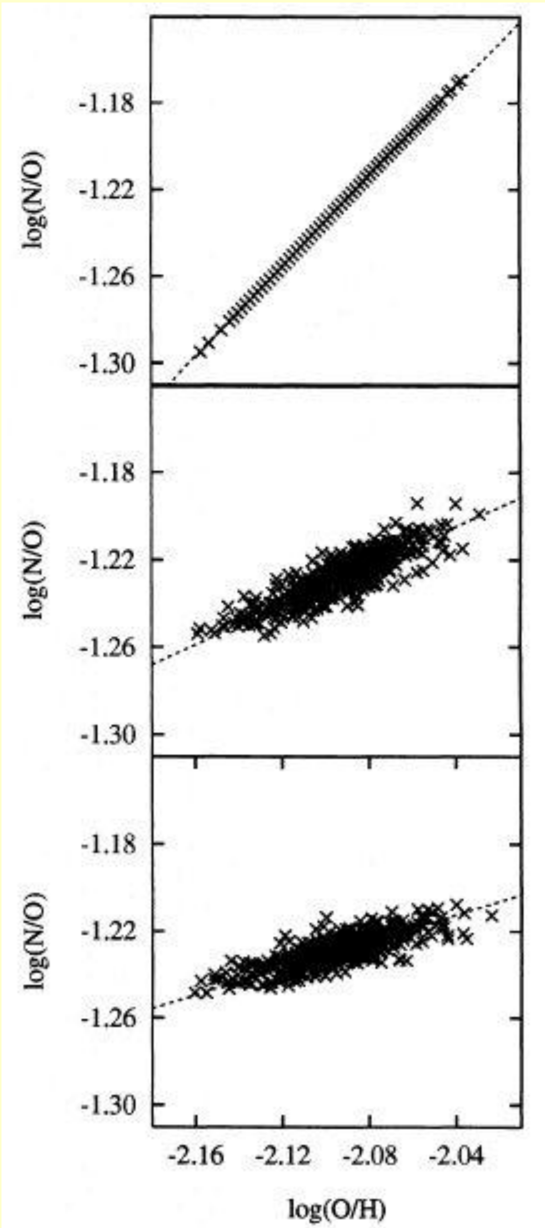
after 2nd time step



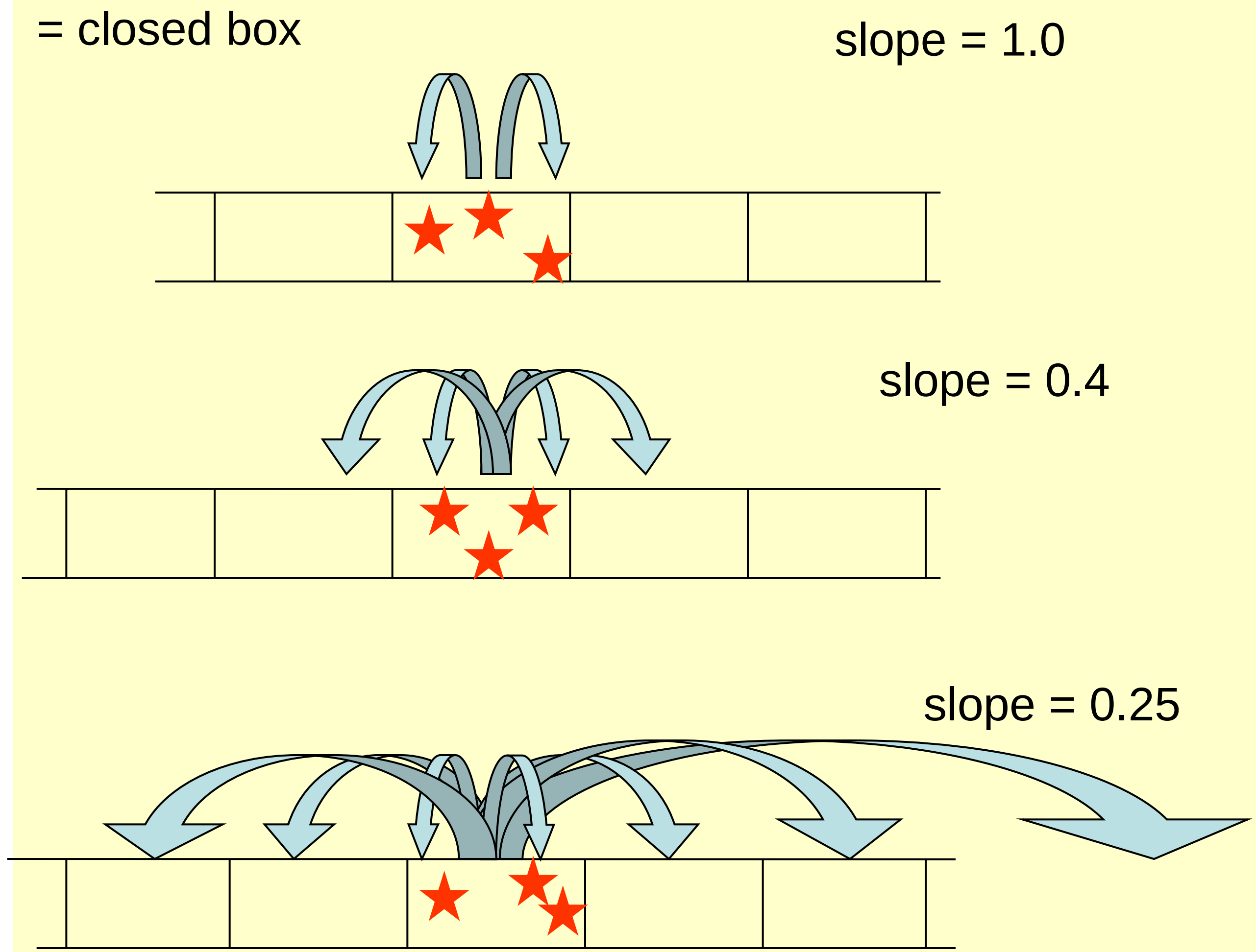
after 3rd time step



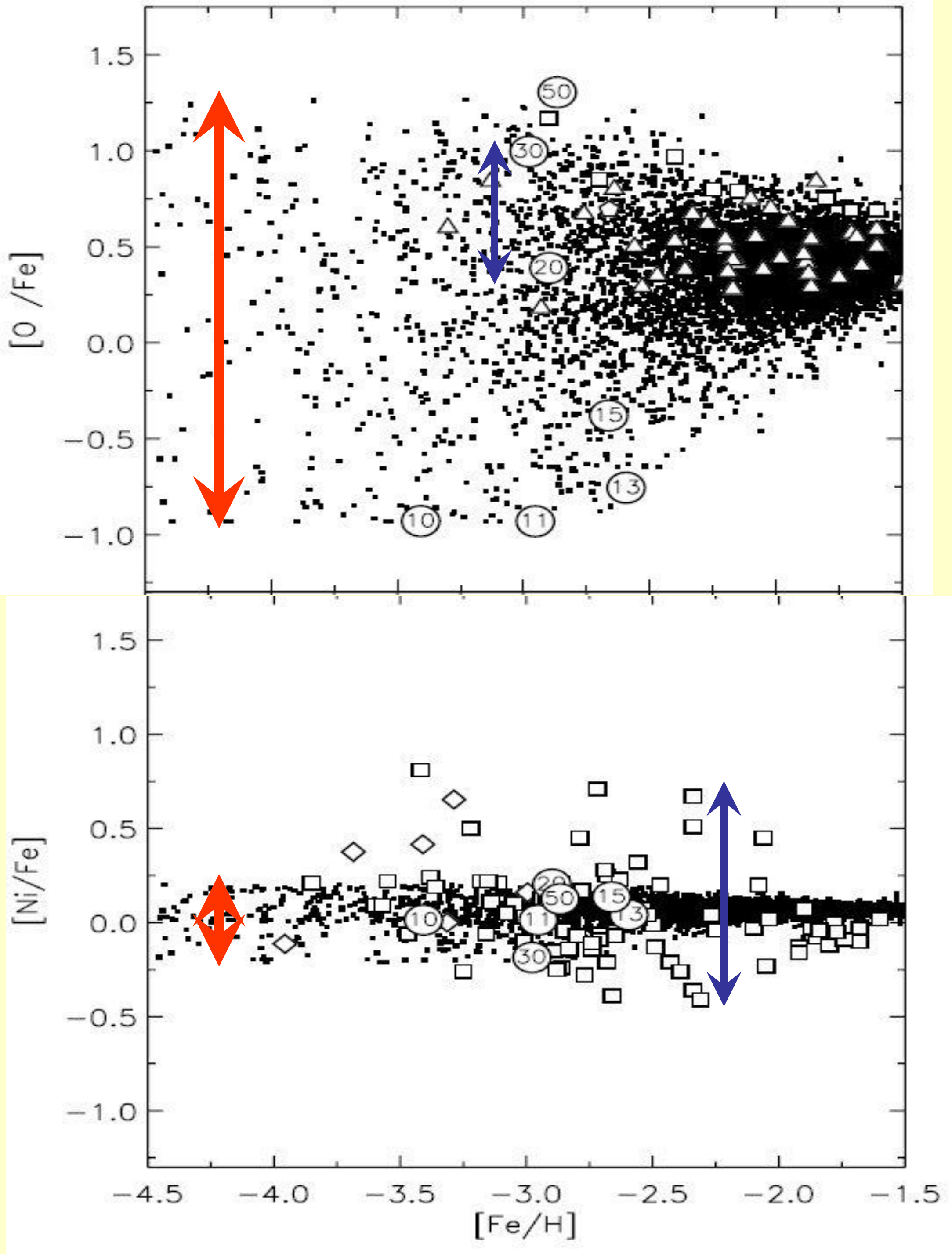
Imperfect mixing: results



Wilmes & Köppen 1995



.. but everything else is like the Simple Model:
mean metallicities, G dwarfs



Imperfect mixing:

halo stars

- gives dispersion of abundances in halo stars

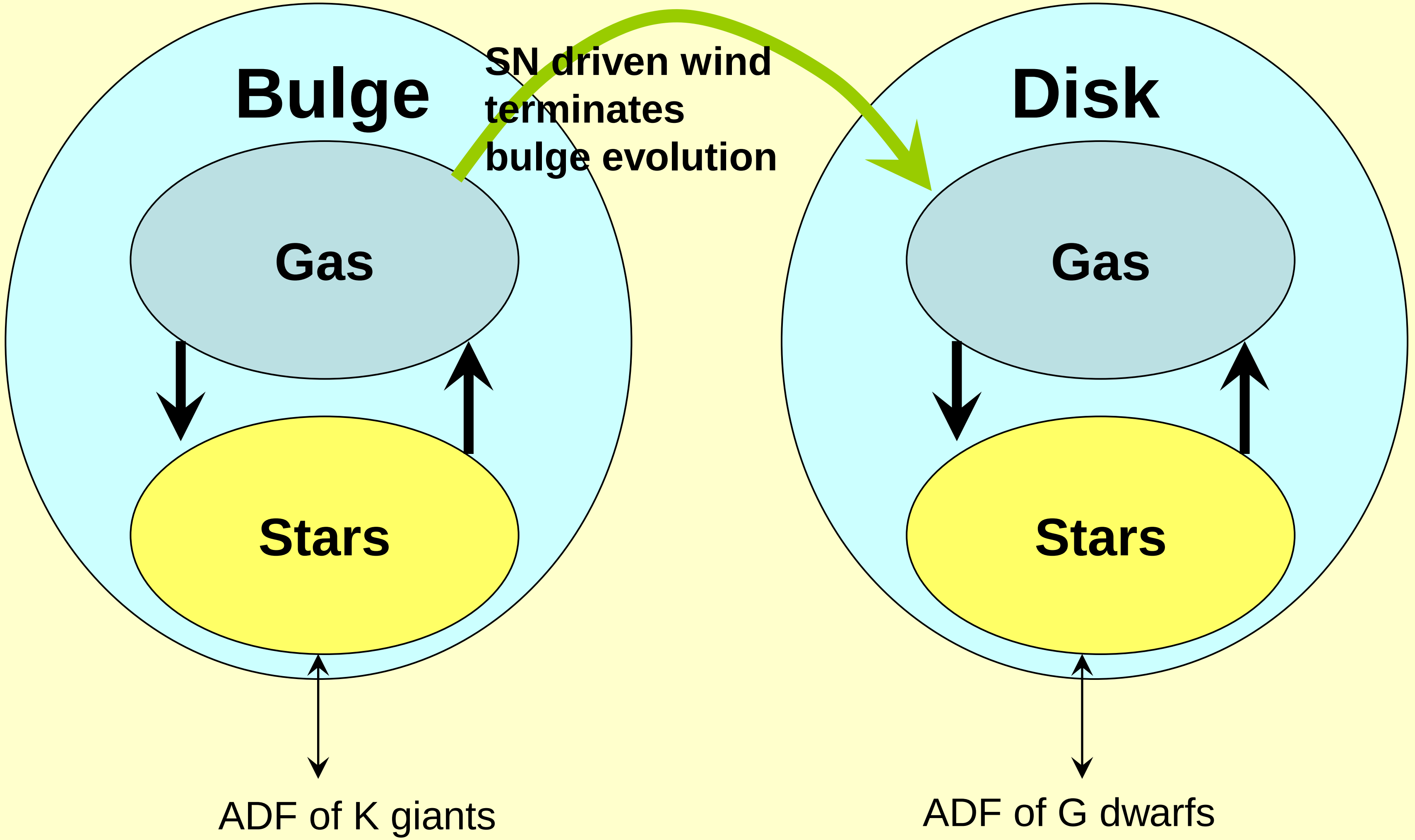
- model dispersions:
  - in O/Fe greater
  - in Ni/Fe smaller

  than observed

➔ nucleosynthesis of massive stars??

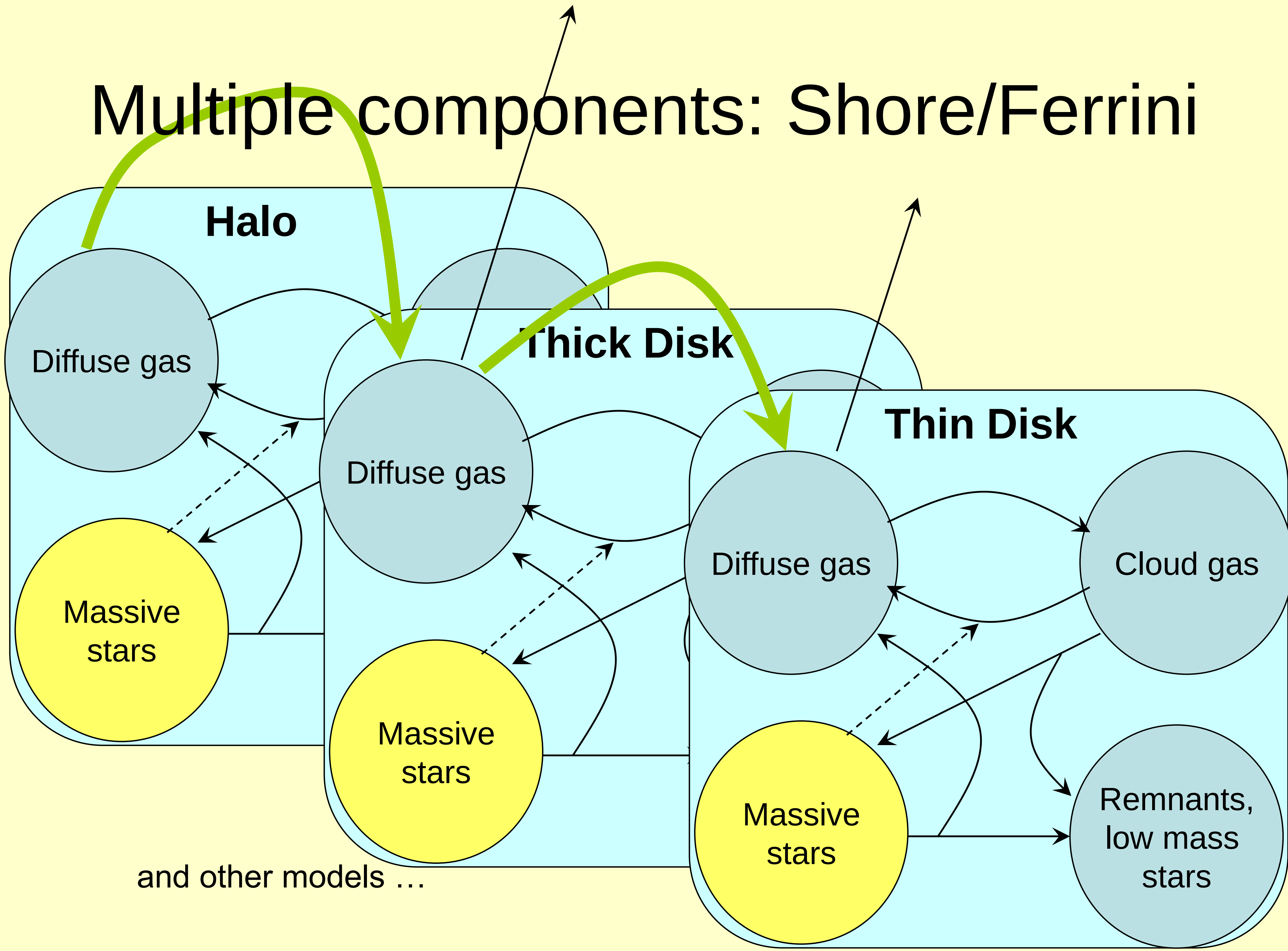
Argast et al. 2000

Multiple components: bulge+disk



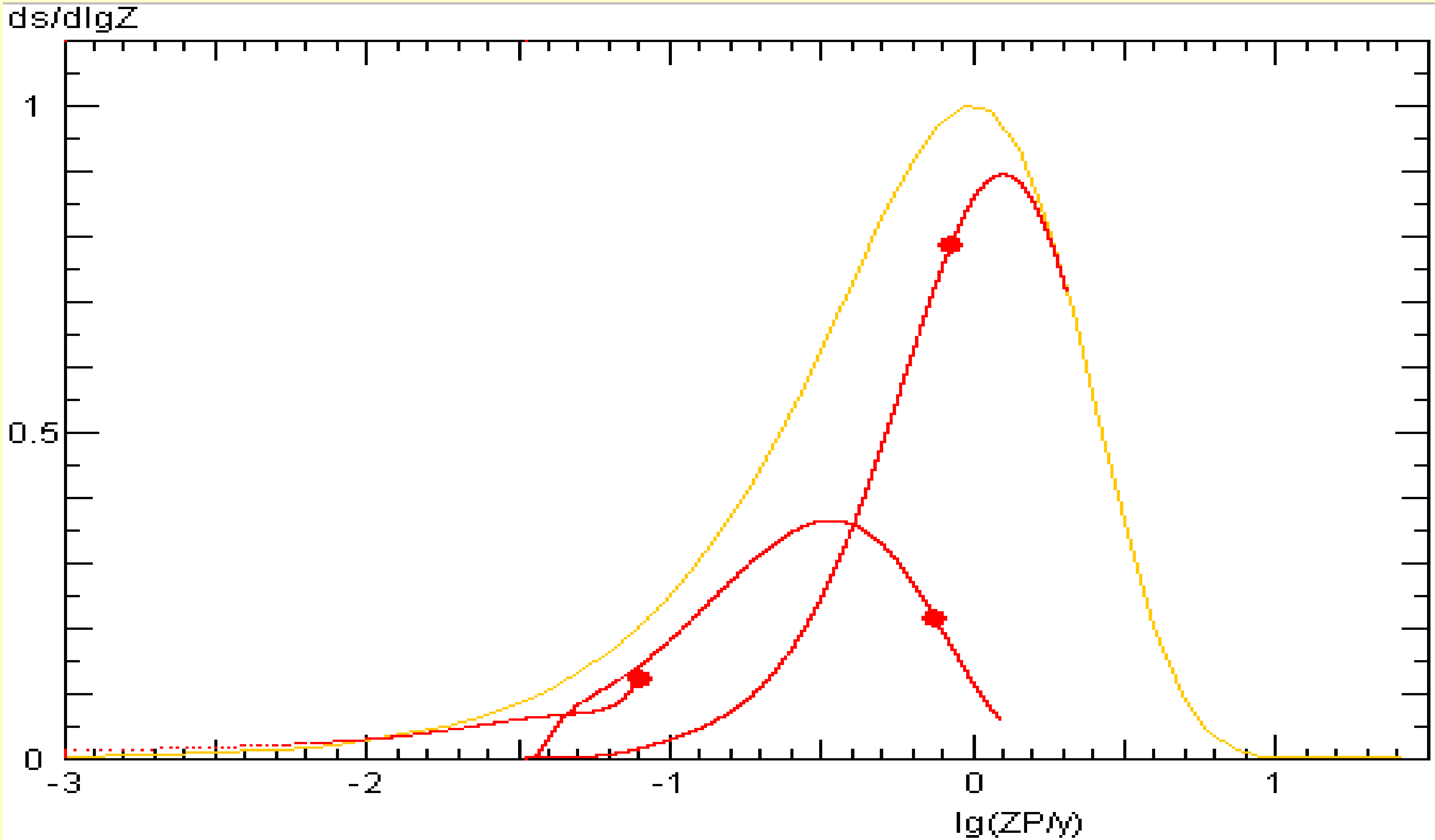
Köppen & Arimoto 1990

Multiple components: Shore/Ferrini



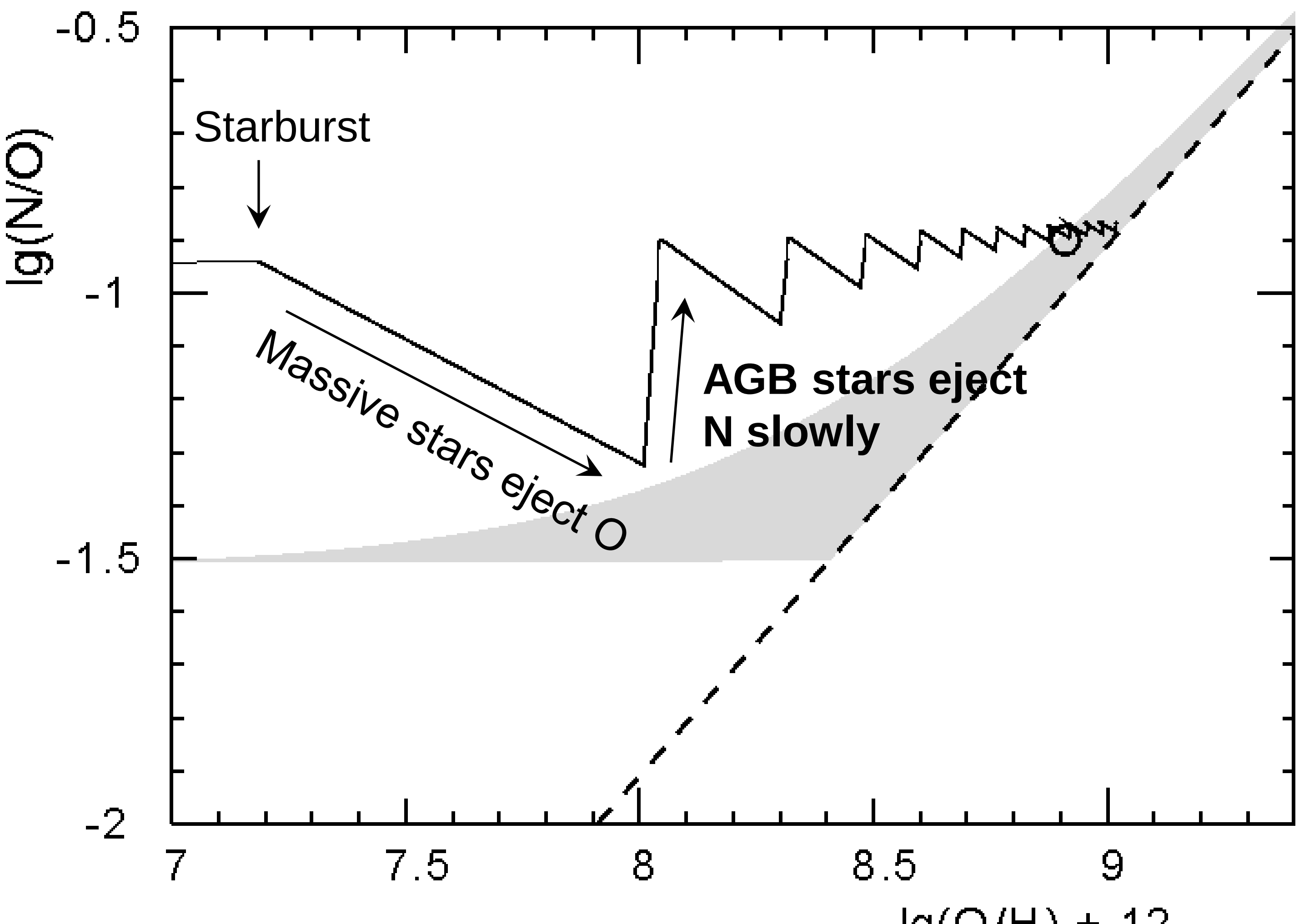
and other models …

➔ADF of halo, thick, thin disk:



Sudden events: Starbursts

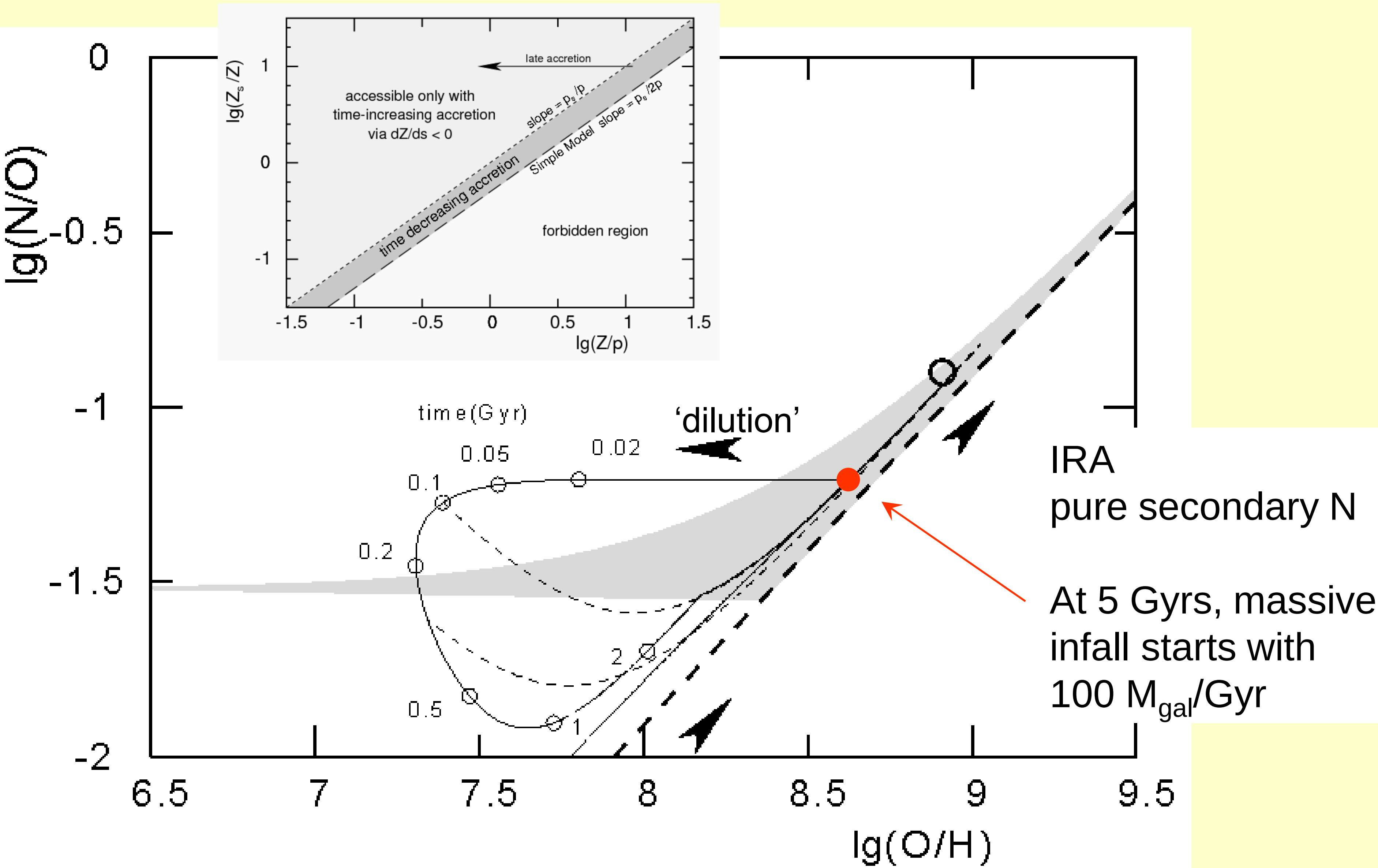
Simple Model is insensitive to SFH (🡅) but it does matter when time-delayed production is involved!



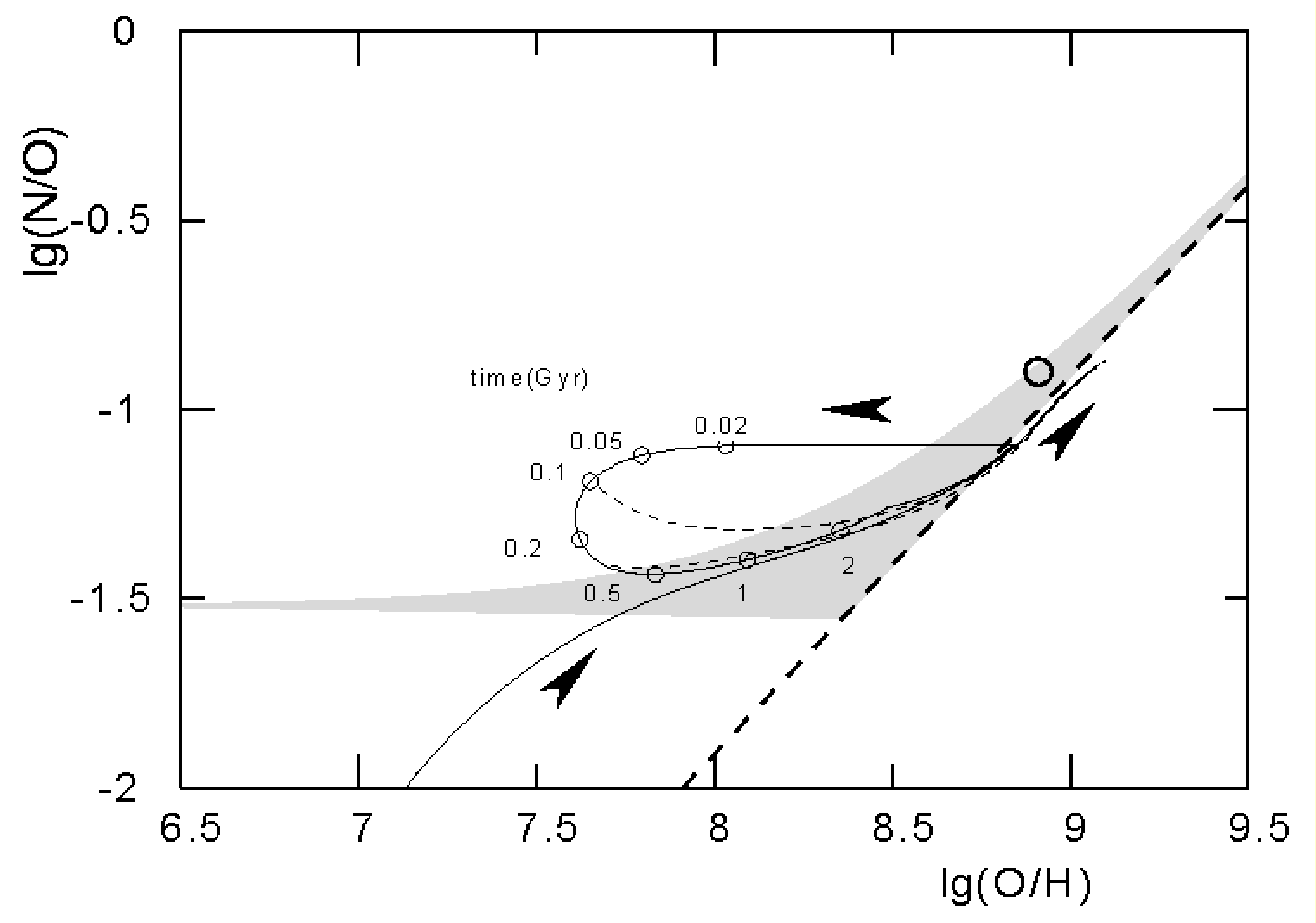
Sudden events: gas removal, outflows

- Galactic winds: accumulated thermal energy (heated by SN) in ISM exceeds gravitational binding energy
- Ram pressure stripping: passage through dense intracluster gas removes ISM
- ➔ Gas + metals escape
- ➔ Star formation stops (truncated ADF)
- Gas could be built up from returns by stars, gas metallicity decreases as the older, metal-poorer stars die …

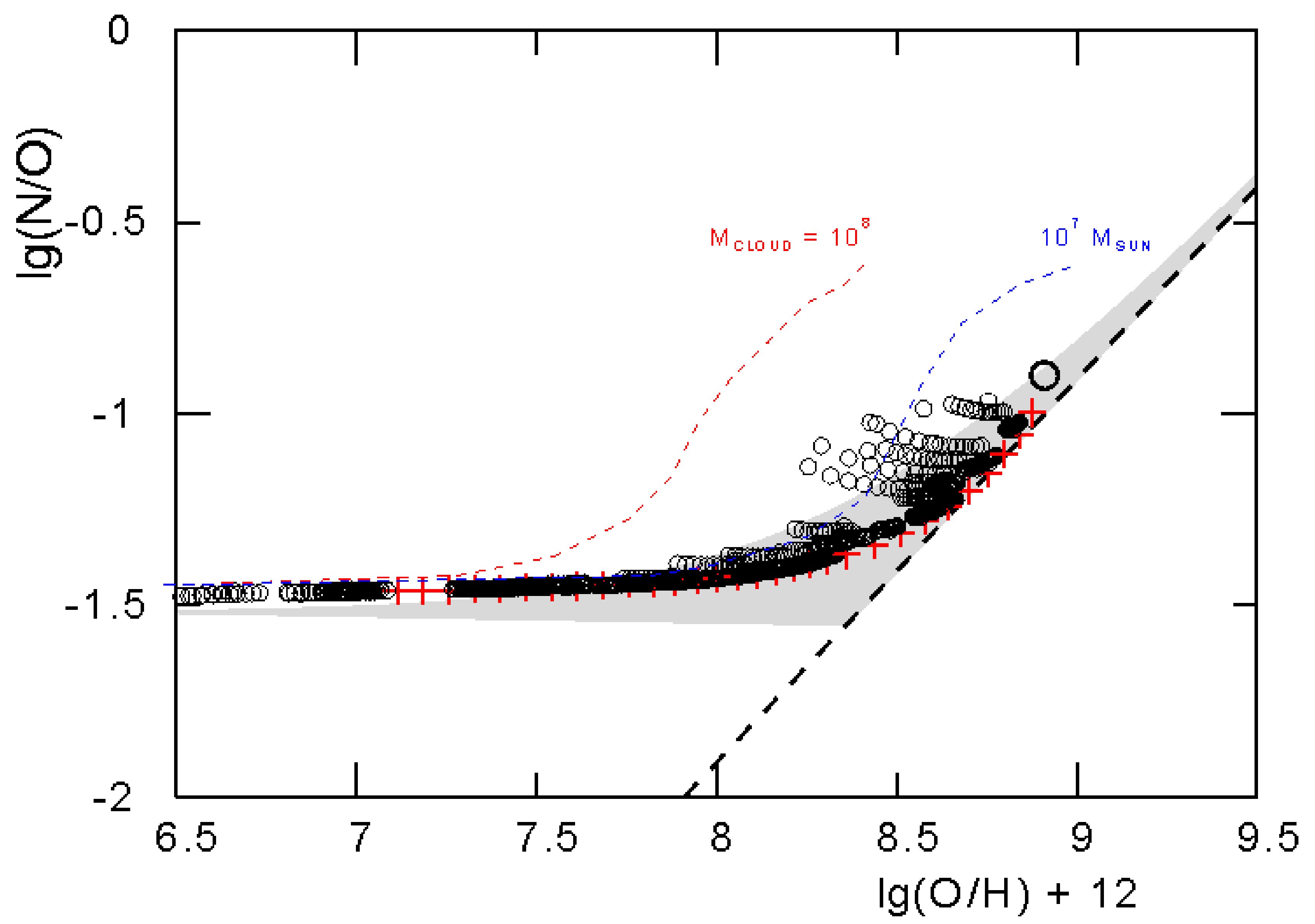
Sudden events: Infall events



Infall events: same model with stellar lifetimes and ‘proper’ nucleosynth. (incl. primary production)



Collisions with High Velocity Clouds …



Köppen & Hensler 2005

The yield

Yield = metal production of a stellar generation

« yield »
« integrated yield »
« IMF weighted yield »

IMF (Salpeter: 1.35)

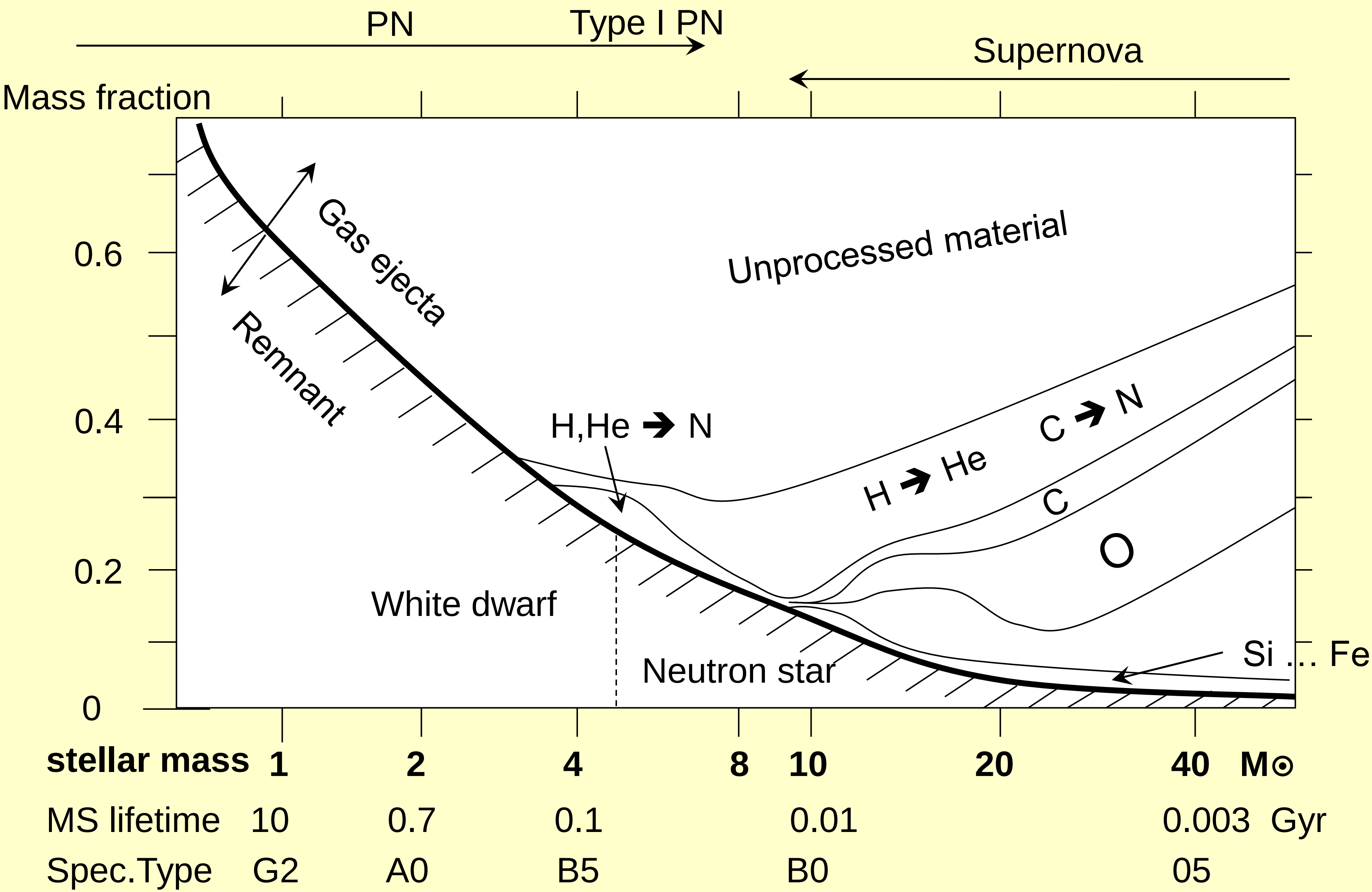
$$y_i = \frac{1}{\alpha} \int_{m_{min}}^{m_{max}} p_i(m) \Phi(m) \, dm$$

locked-up mass fraction

max (IMF lower mass limit, turn-off mass)

« stellar yield »
= <u>Fraction of stellar mass ejected in form of newly produced element *i*</u>
« fresh stellar yield »
« net stellar yield »
… !!!

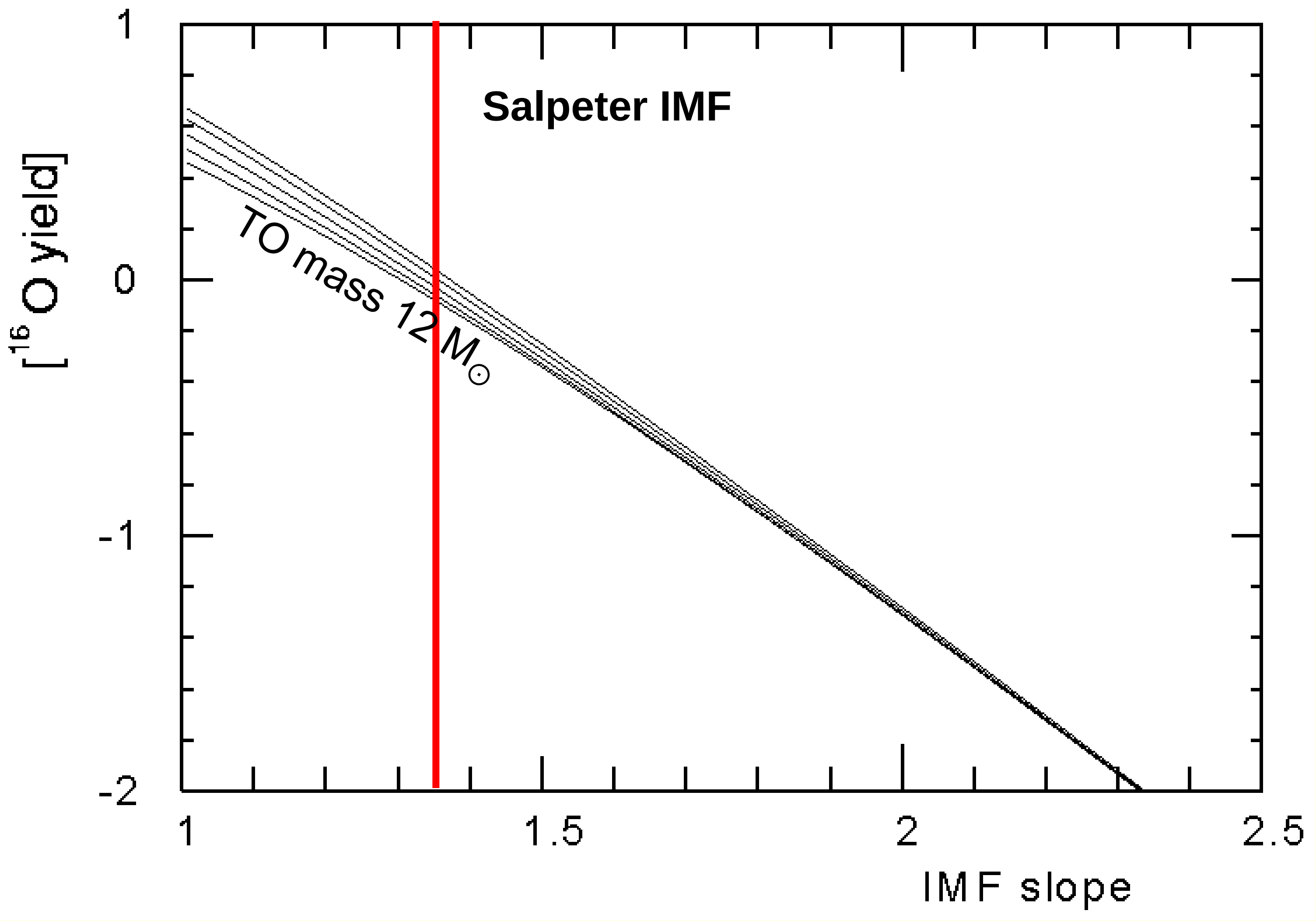
Stellar yields: what stars produce:



Accuracy of stellar yields and the yield ?

Generally not better than a factor of 2 …

Slope of IMF is very important:

- 

Effective yields

- We may use the Simple Model relations to define « effective yields »:
- Gas metallicity: $y_{eff} = -Z/\ln f$
- Stellar ADF
  - slope $yeff = -1/(\Delta \ln(dn/dZ)/\Delta Z)$
  - peak in plot as dn/dlogZ

E.g. oxygen

H ⟶ He ⟶ C ⟶ O

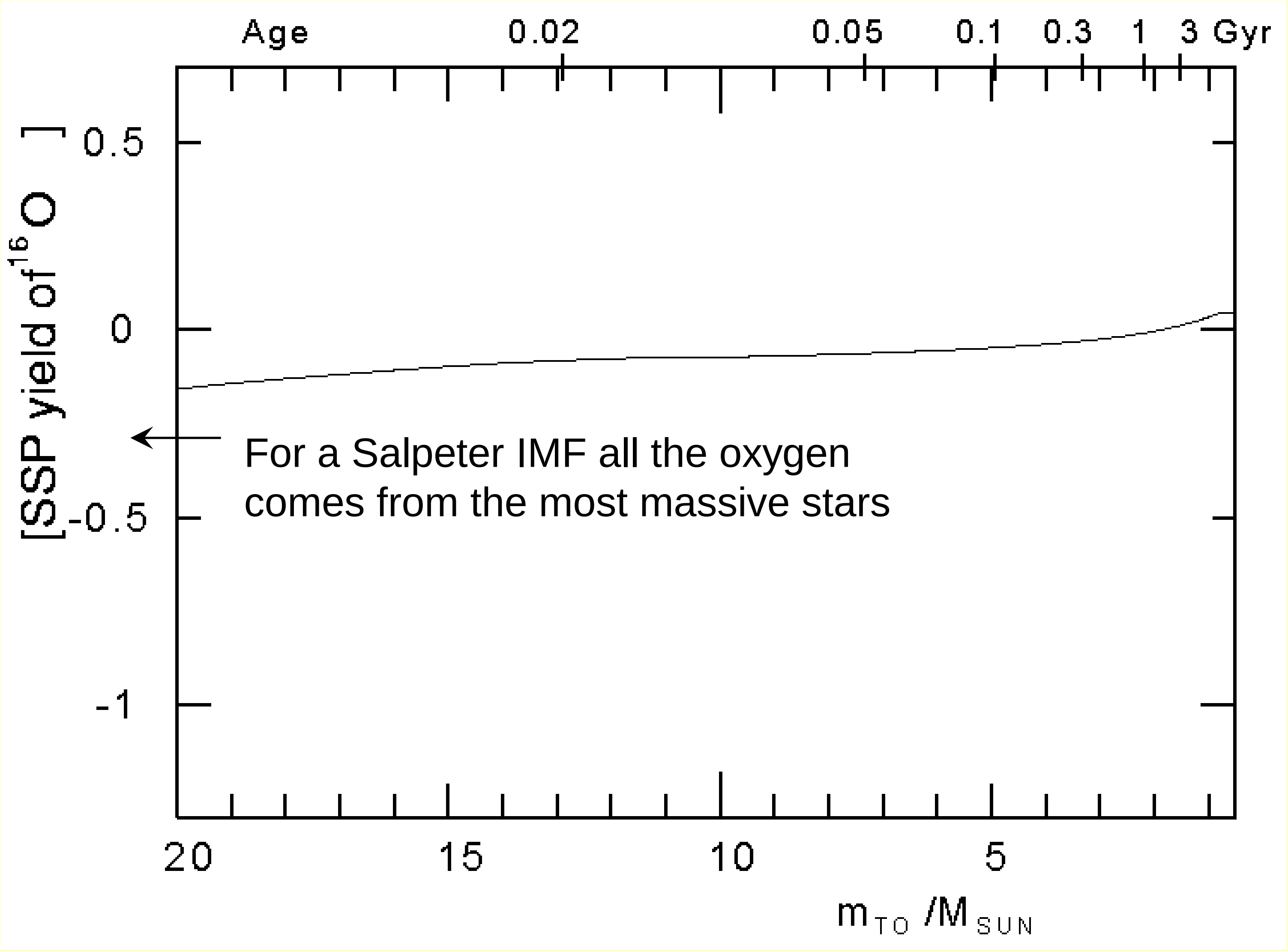
CNO cycle — triple α — α addition

Main sequence
(H-burning)

post-main sequence
(He-burning)

From H to O in one stellar generation
➔ yield independent of metallicity: H >> O
➔ ‘primary’ production

Single Stellar Population Yield



How to make nitrogen …

1) H → He → C

CNO cycle — **Main sequence**

triple α — post-main sequence

ISM

2) C → N

CNO cycle
Main sequence

Takes two stellar generations
➔ yield $\propto$ metallicity
➔ ‘secondary’ production

… but it also works this way …

H ⟶ He ⟶ C

CNO cycle
Main sequence

triple α
post_main sequence

3rd dredge-up on AGB

C ⟶ N

CNO cycle (hot-bottom burning)
AGB

All in the same star
➔ yield independent of metallicity
➔ ‘primary’ production

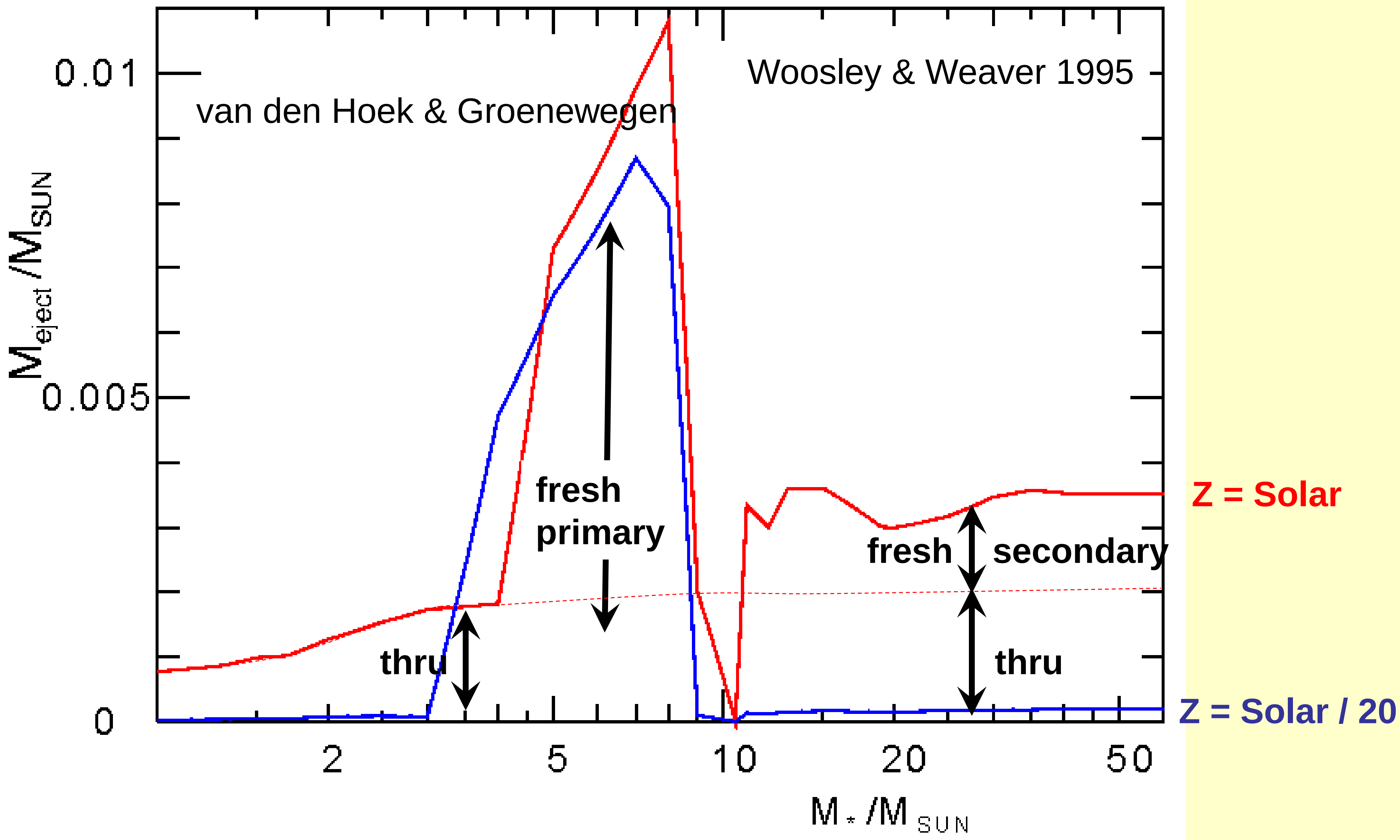
Intermediate mass stars

- mass range: 3 … 9 $M_\odot$

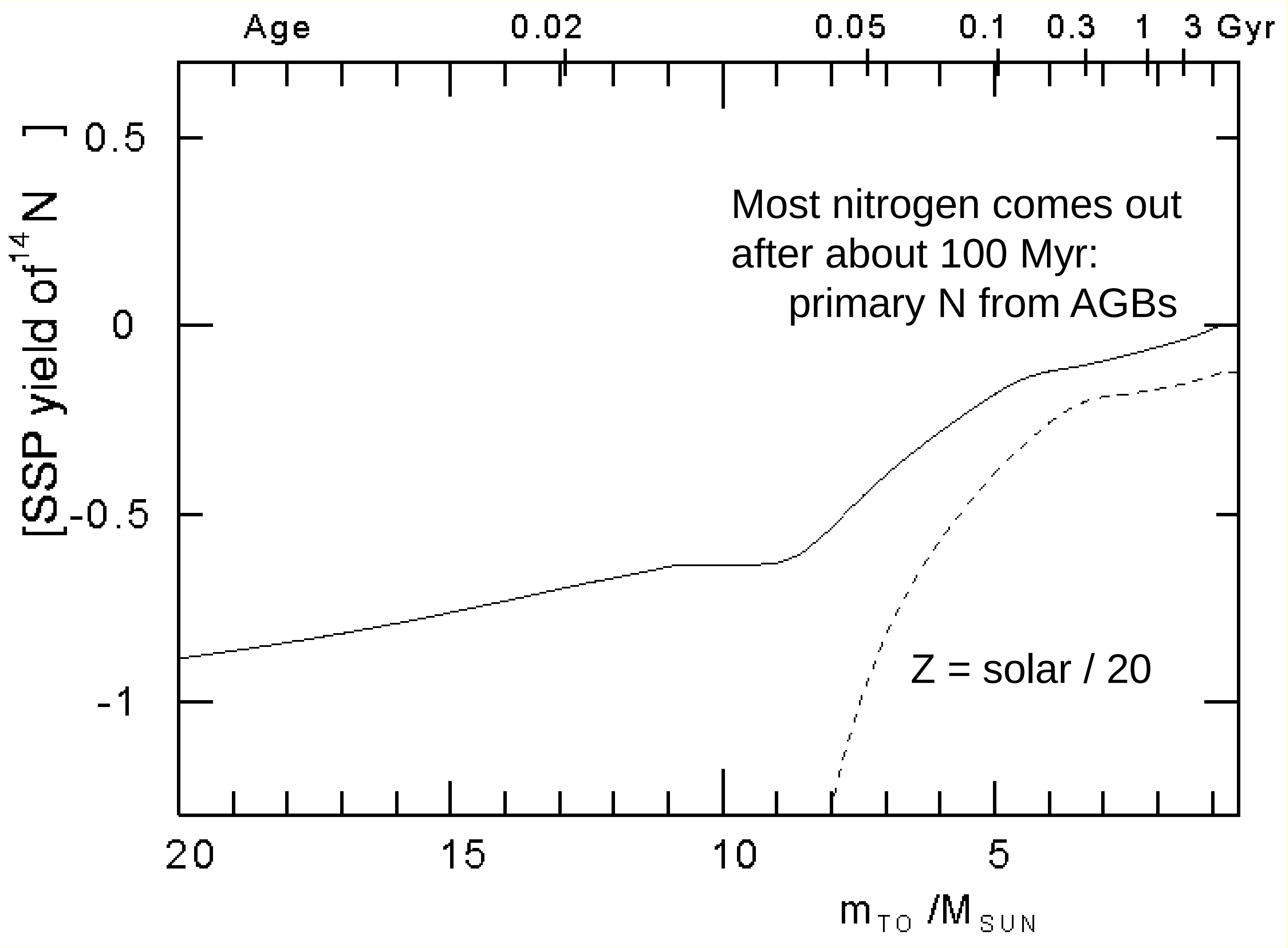
- produce He, C, N

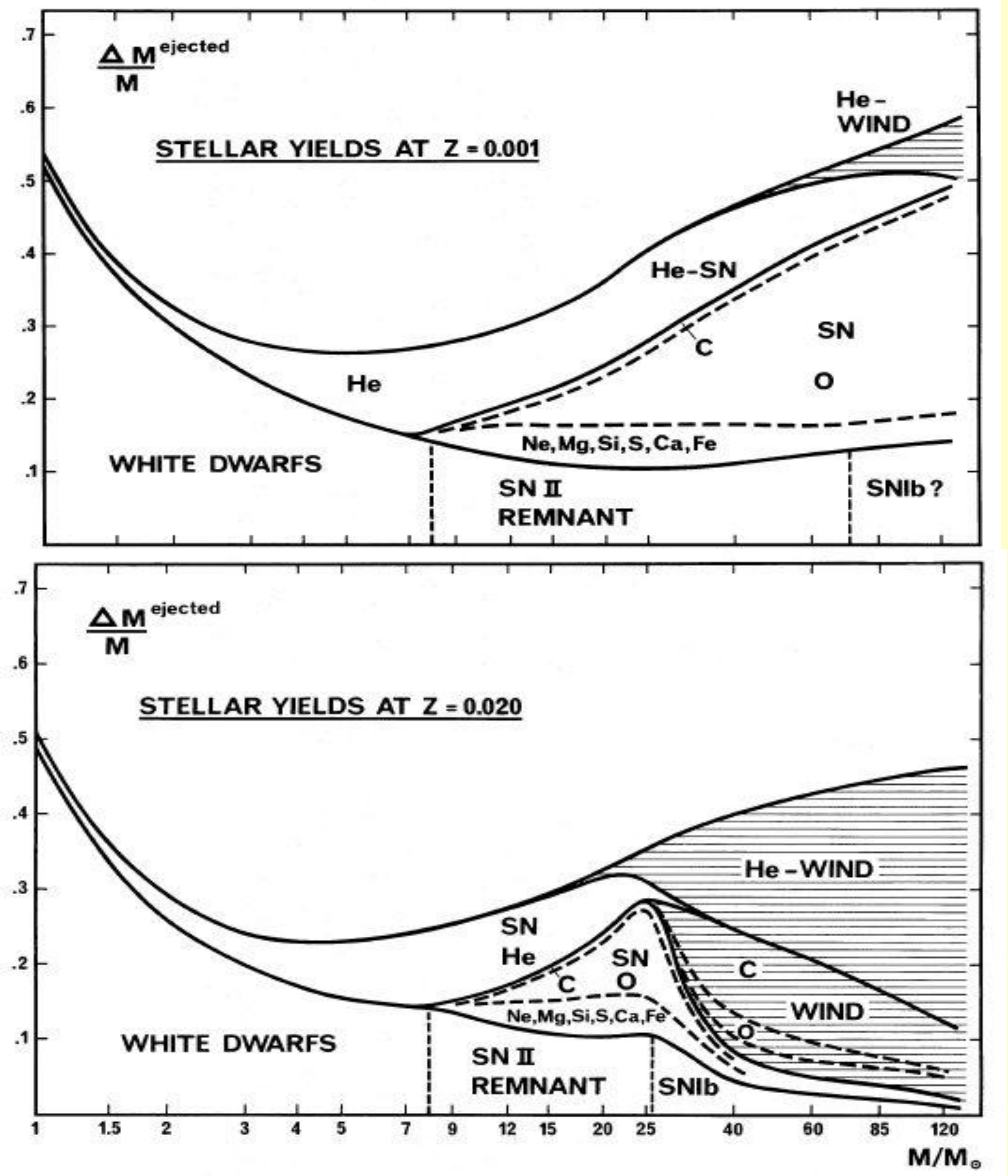
- complex evolution:
  - convection + ‘overshooting’ (no exact theory, but can be scaled from numerical HD simulations)
  - internal mixing by dredge-ups on RGB, AGB
  - thermal pulses (AGB) ➔ full models very scarce
  - mass-loss (RGB, AGB, no theory)
  - details of final phases, ejection of PN

Ejected Nitrogen = fresh + thru



SSP yield for nitrogen

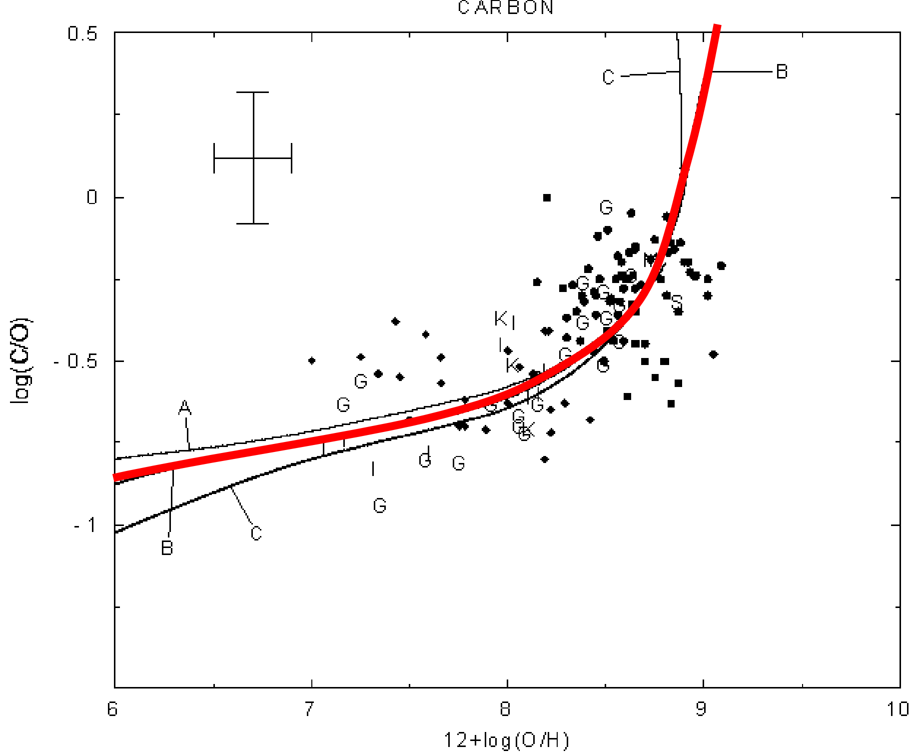




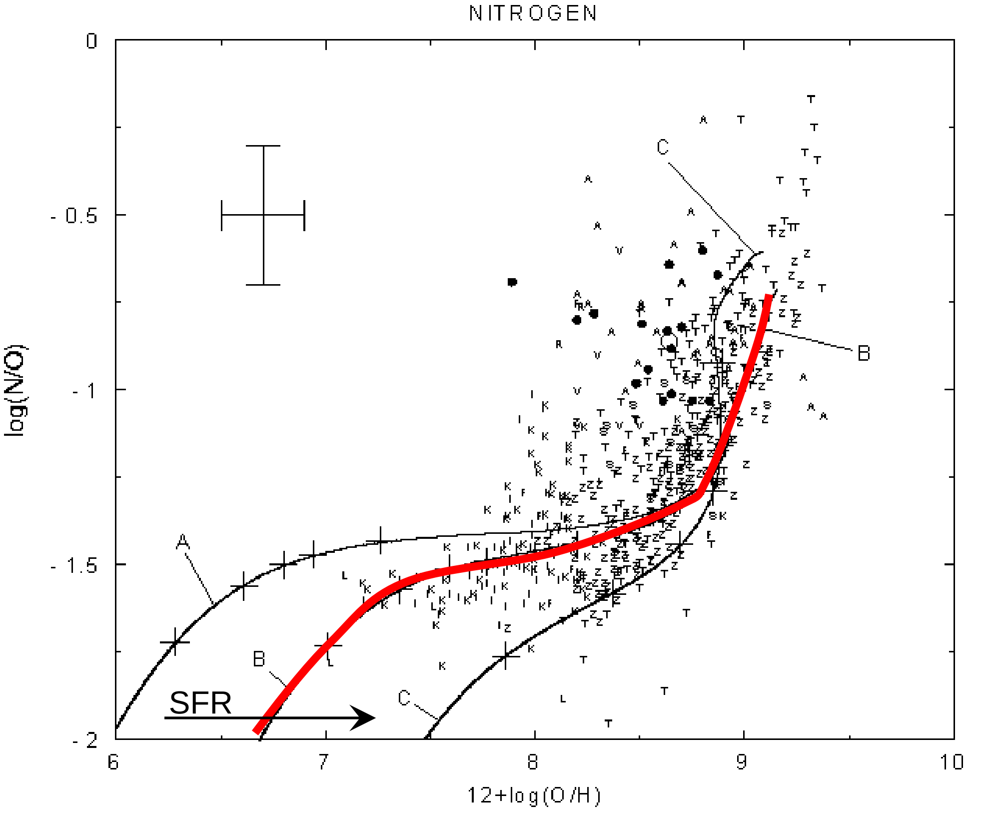
Maeder 1992:

Lower O-yields from metal-rich massive stars due to higher mass loss from stellar wind

➔ Oxygen is NOT « primary »

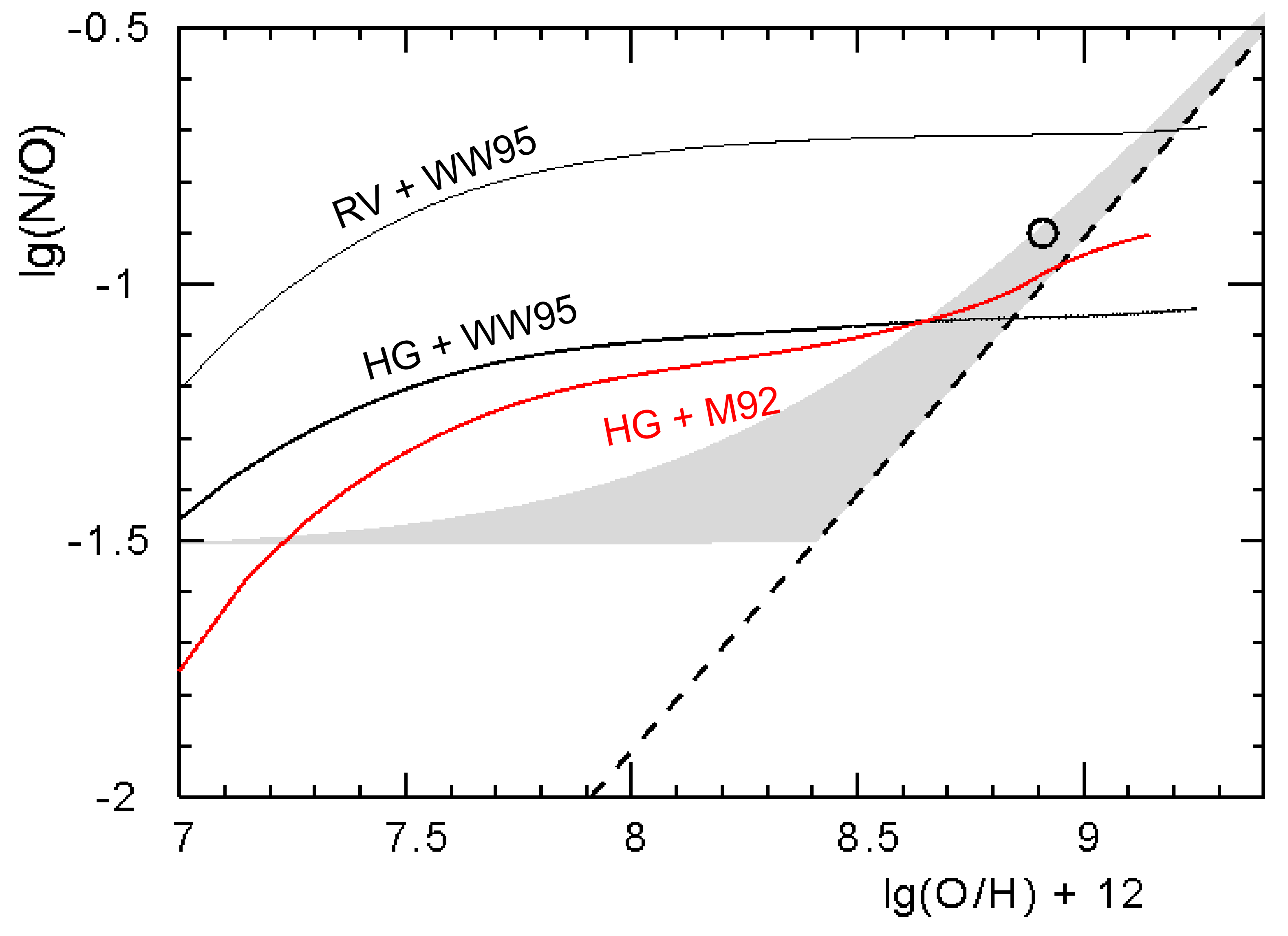


Reduction of O yield by stellar winds (Maeder 92) can explain the increase of both C/O and N/O

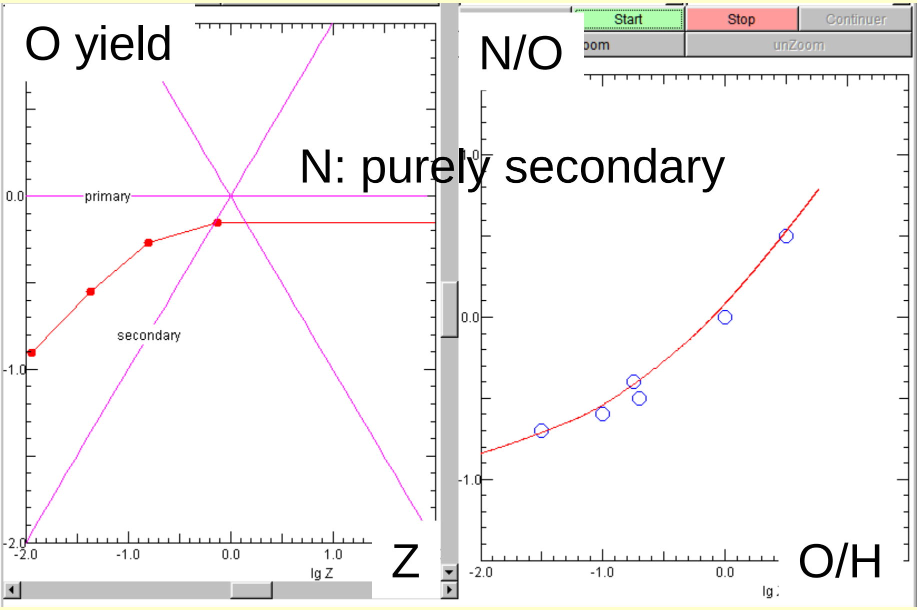


Henry, Edmunds, Köppen 2000

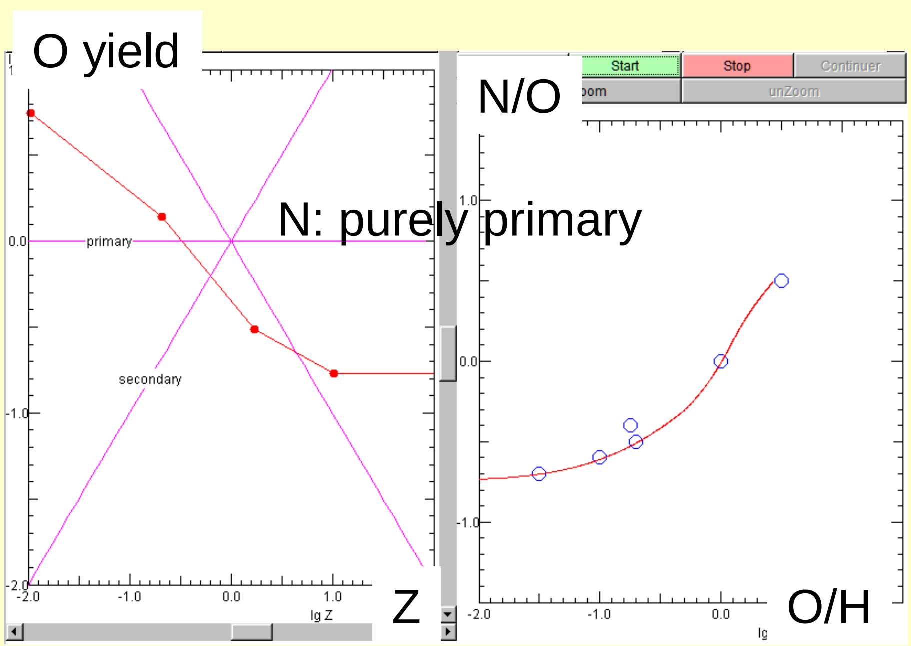
Illustrative closed box models

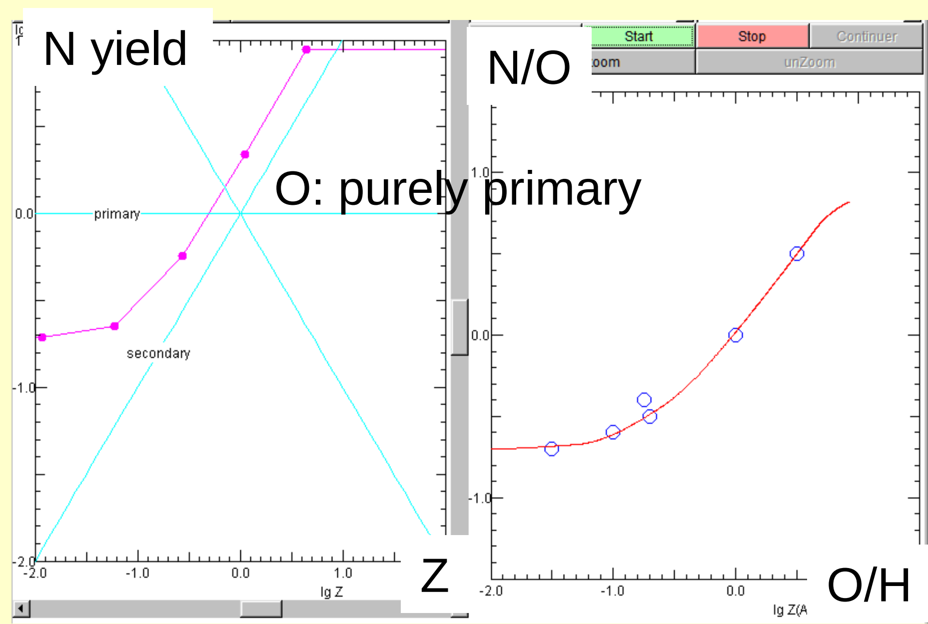


RV = Renzini&Voli 1981
HG = van den Hoek&Groenewegen 1994
WW95 = Woosley&Weaver 1995
M92 = Maeder 1992

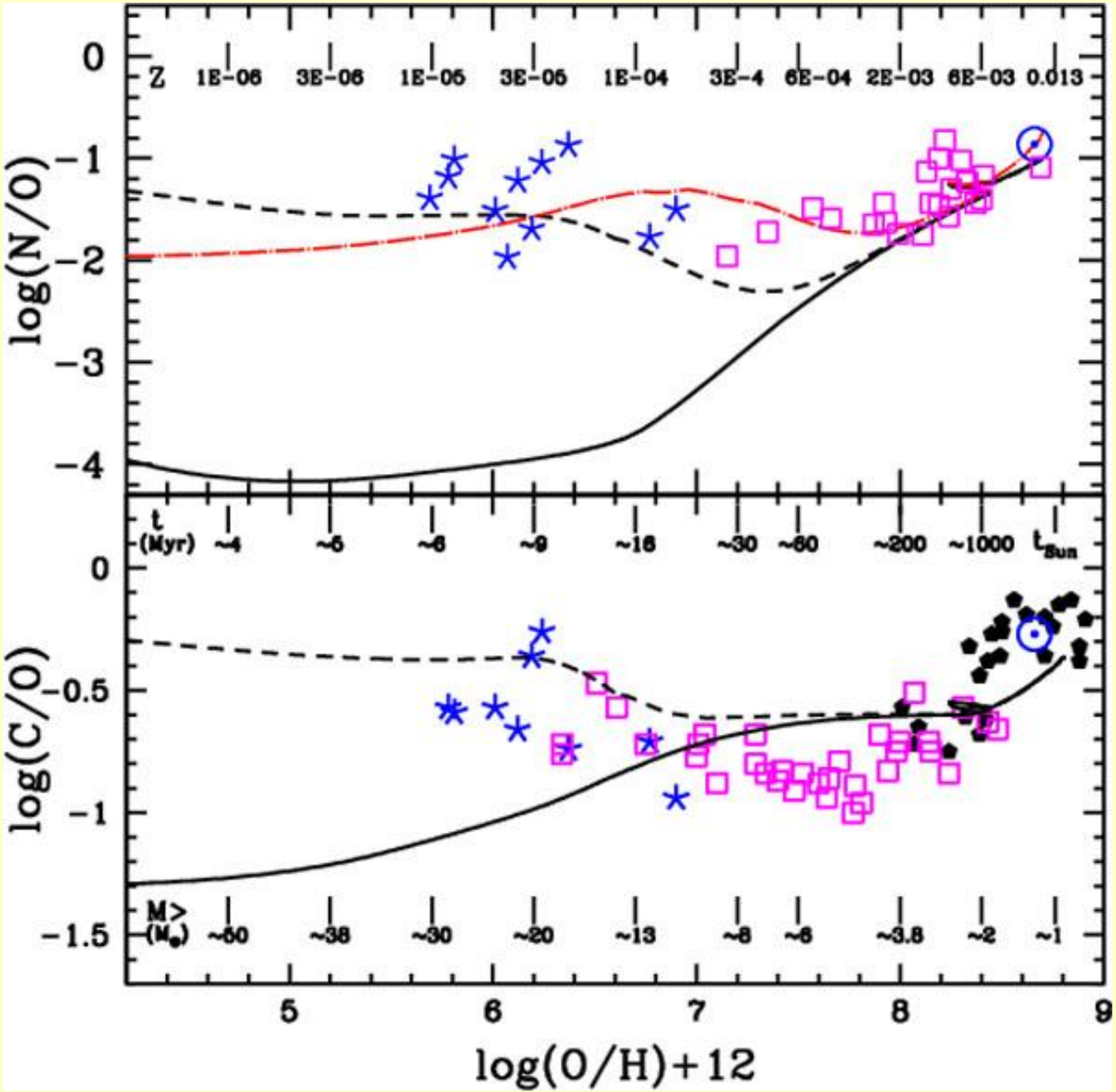


The observed NO-OH relation can well be matched by different assumptions on the O and N yields' metallicity dependence …





The last word on N/O so far …

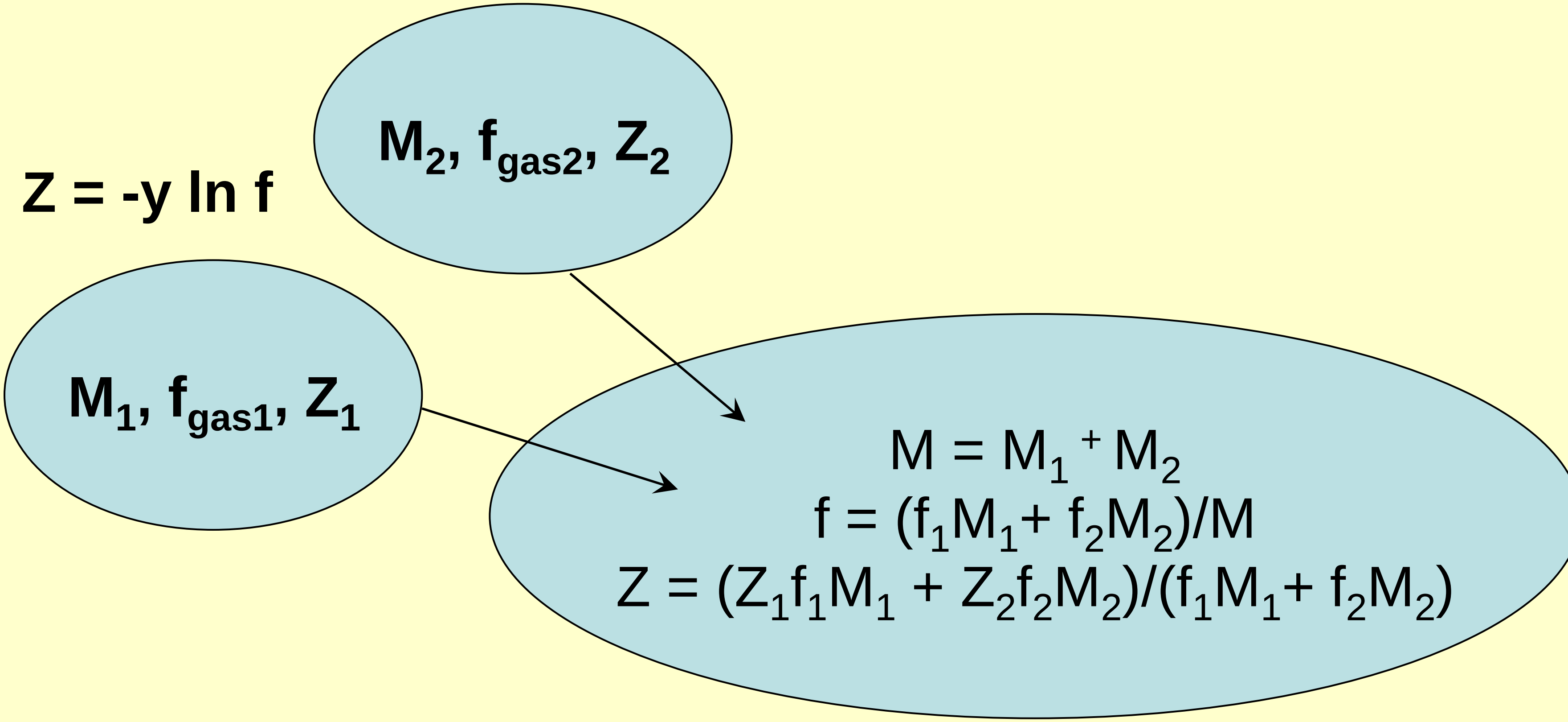


- * * * Spite² (2005) find high N/O and C/O ratios in very metal poor stars
- Implies large N-production before AGB stars die ➔ massive stars must produce large C+N
- Hirschi (2007) rapidly rotating massive stars produce substantial C+N
- 🡐 Chiappini (2006) chem.ev. models - - - -

Usefulness of the Simple Model

- Simple Model is the most efficient way to make metals while consuming as little gas as possible
- Inflow of gas, outflows of gas or ejecta only **decrease** the effective yield
- Gives an upper limit for metallicity and hence the true yield
- $Z_p = -y_p \ln f$ and $Z_s/Z_p = y_s/2y_p * Z_p$ are insensitive to SFR, SFH, (infall) …

Multiple components: Mergers



Example: $M_1 = M_2$, $f_1 = 0.1$, $f_2 = 0.9$ ➔ $Z_1/y = 2.3$, $Z_2/y = 0.1$

➔ $f = 0.5$, $Z/y = 0.32$ but $-\ln(0.5) = 0.69$

➔ merged system is **only a factor 2** off the Simple Model prediction