Introduction to Radioastronomy:

Physical Processes

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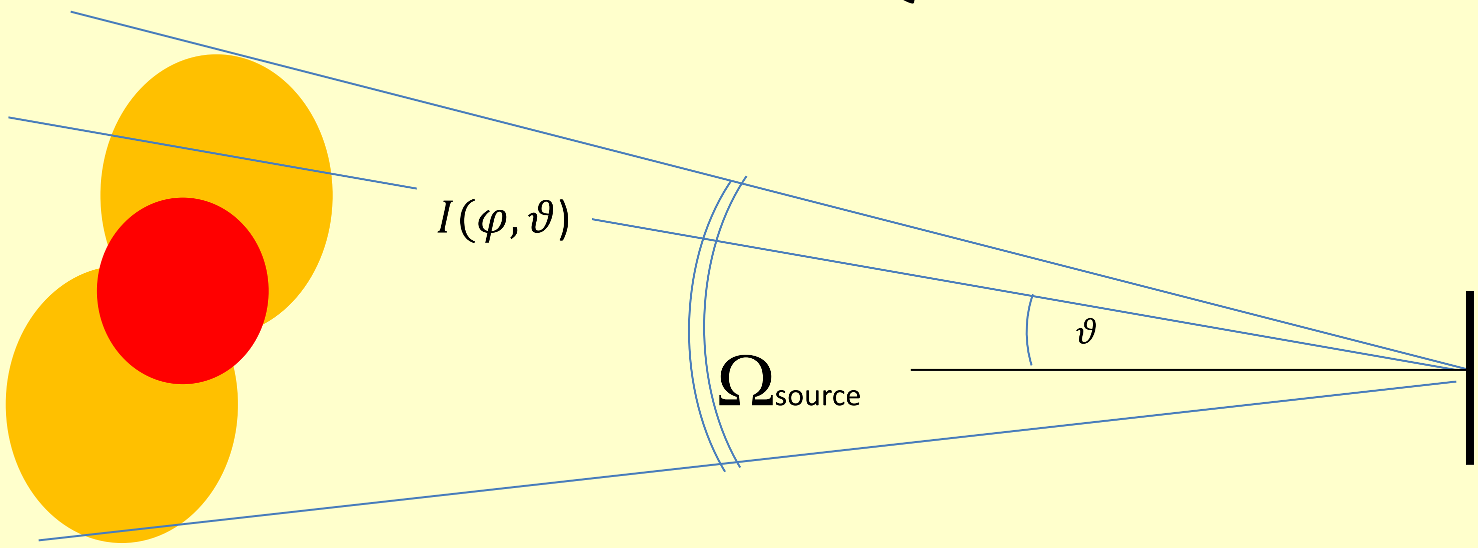
Radiation Quantities

- (specific) **Intensity** = energy flow per unit time, per unit area, per unit frequency, **into a certain direction, and per unit solid angle** [also used: Brightness, Surface brightness]; independent of distance if no extinction or refraction
- **Flux** = energy flow per unit time, per unit area, per unit frequency, and **perpendicular through the test area** [also: Flux density]; decreases with distance^2 from source

Radiation Quantities



Flux received from an extended source:

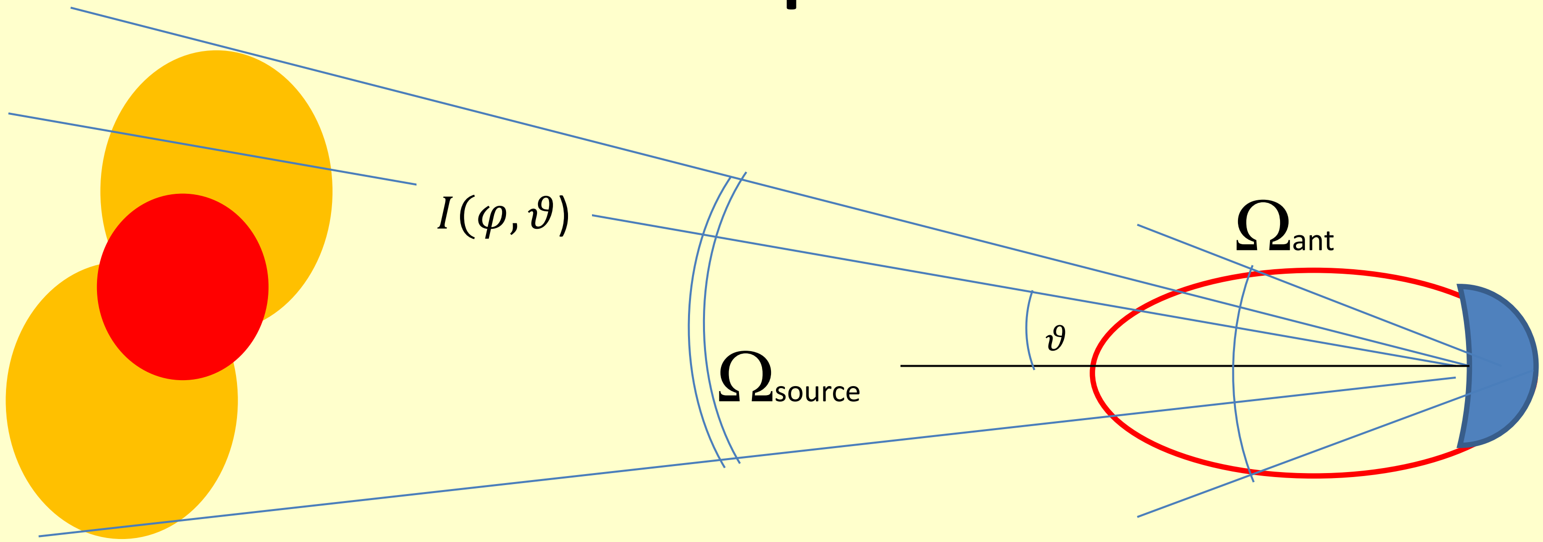
$$F = \int I(\varphi, \vartheta)\ \cos\vartheta\ d\varphi d\vartheta$$

'antenna pattern' of test area

homogeneous source: $F = I * \Omega_{source}$

solid angle of spherical source: $\Omega_{source} = \pi r^2 / d^2$

The antenna pattern matters



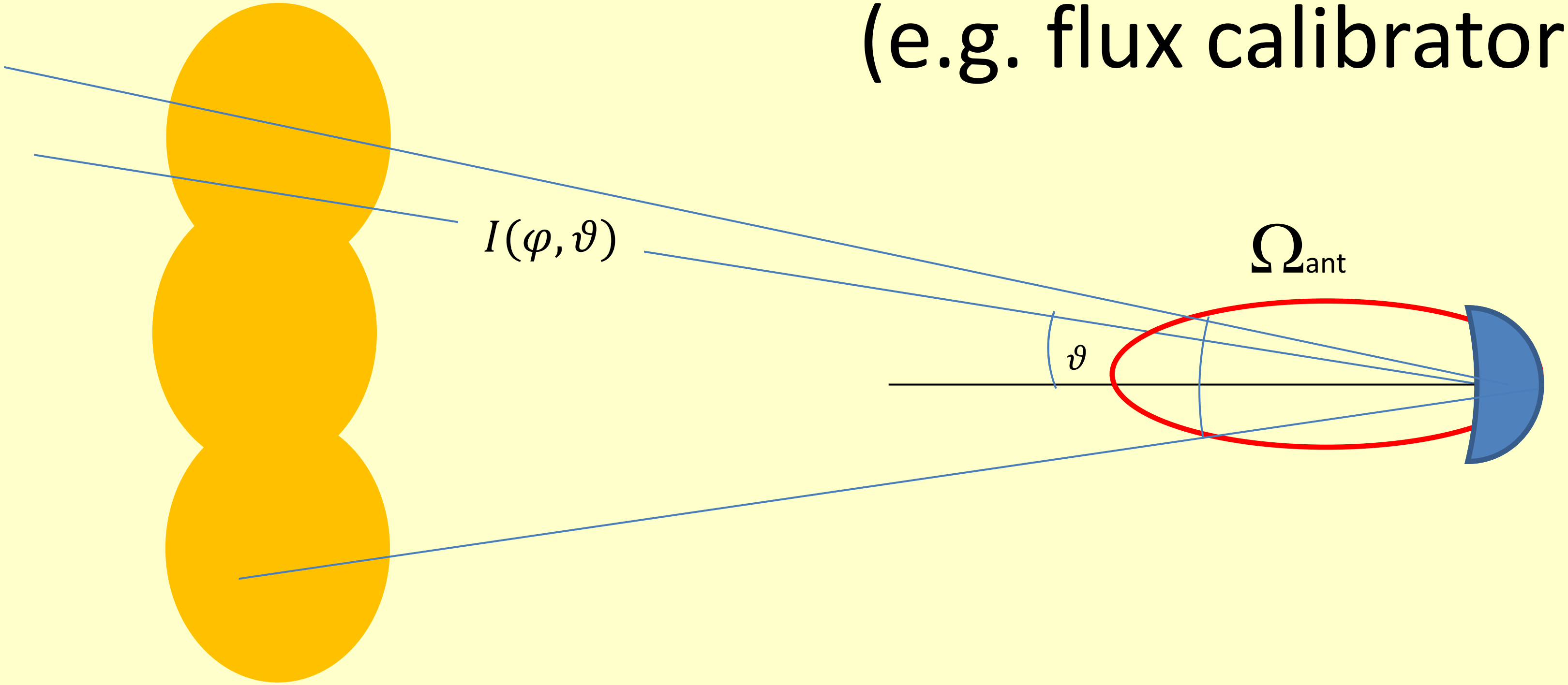
Flux received from source:

$$F = \int I(\varphi,\vartheta)\ A(\varphi,\vartheta)\ d\varphi d\vartheta$$

antenna pattern solid angle Ω_{ant}

source brightness distribution: Ω_{source}

Isotropic source $(\Omega_{source} \gg \Omega_{ant})$ (e.g. flux calibrator)



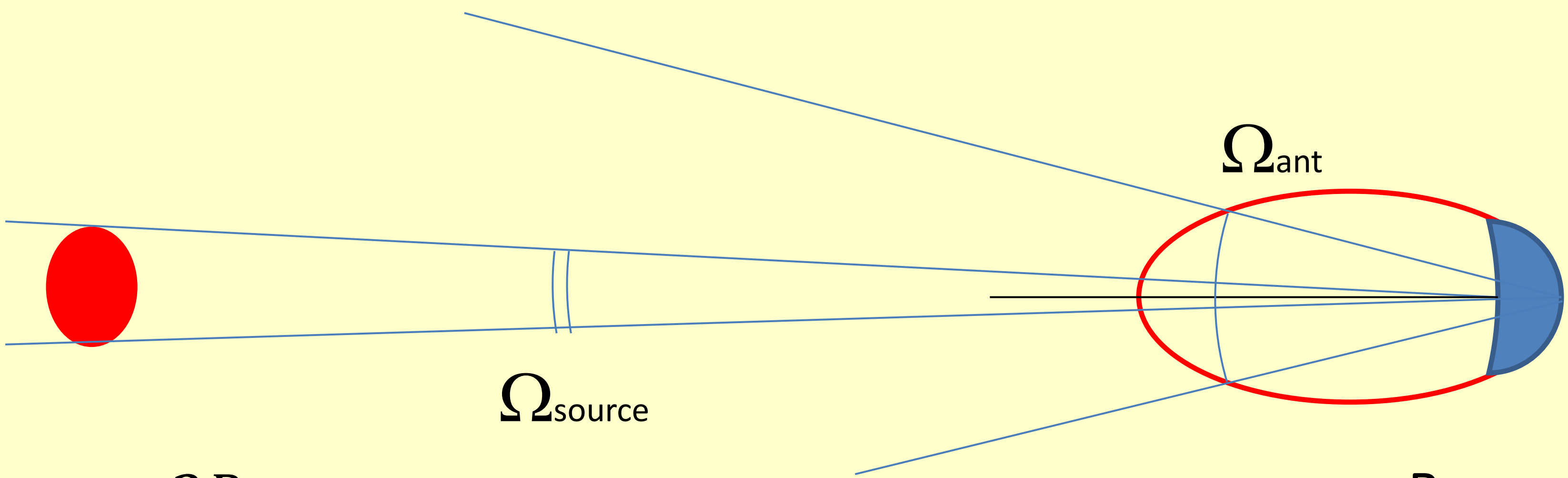
$$\frac{2P}{A_{eff}} = F = I(T_B) * \int A(\varphi,\vartheta)\, d\varphi d\vartheta = I(T_B) * \Omega_{ant}$$

$$\frac{2k\, T_A}{\lambda^2/\Omega_{ant}} = \qquad = \frac{2k\, T_B}{\lambda^2} * \Omega_{ant}$$

$$\boxed{T_A = T_B}$$

antenna temperature — brightness temperature of source

Point source ($\Omega_{source} \ll \Omega_{ant}$)



$$\frac{2P}{A_{eff}} = F = A(\vartheta \approx 0) * \int I(\varphi,\vartheta)\,d\varphi\,d\vartheta$$

Beam centre:
$A(\vartheta \approx 0) = 1$

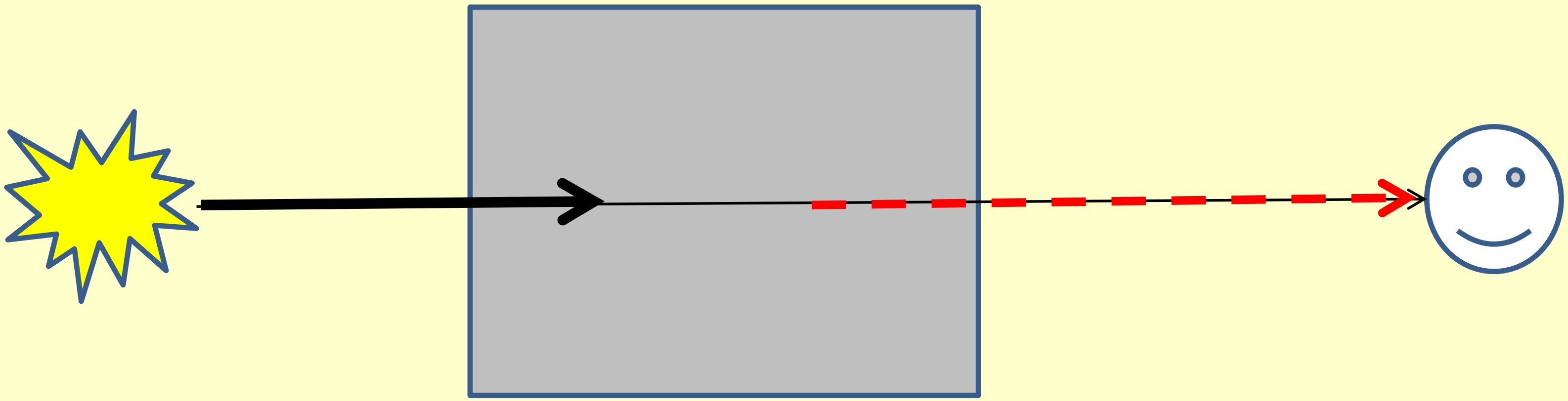
$$\frac{2k\,T_A}{\lambda^2/\Omega_{ant}} = \ I(T_B) * \Omega_{source} = \frac{2k\,T_B}{\lambda^2}\Omega_{source}$$

$$\boxed{T_A = T_B\ \frac{\Omega_{source}}{\Omega_{ant}} \leq T_B}$$

Radiation and Matter

- Emissivity = energy production per unit time, unit volume, unit frequency
- Absorption coefficient (Opacity) = fraction of the incident radiative power absorbed, per unit volume, unit frequency

Radiative Transfer (I)



- Intensity is modified by absorption and emission:

$$\frac{dI}{ds} = -\kappa\, I + j$$

Indices of F, I, κ, j mark whether they are taken to be per unit frequency or per unit wavelength interval …

Radiative Transfer (II)

- Formal solution (planar slab)
  - **Optical depth** (observer at s=0)

    $$d\tau = -\,\kappa\, ds$$

    solution: $\tau(s) = \int_0^s \kappa(s')\, ds'$
  - Intensity

$$I = I_0\, e^{-\tau(L)} + \int_0^L j(s')\, e^{-\tau(s')}\, ds'$$

$$= I_0\, e^{-\tau(L)} + \int_0^L S(s')\, e^{-\tau(s')}\, d\tau(s')$$

Source function S = j/κ = B(f, T(s)) (in TE)

Radiative Transfer (IV)

- Homogeneous slab, **optically thick** limit:
  $\tau(L) >> 1$:

$$I = 0 + S * \int_0^L e^{-\tau}\, d\tau = S\left(1 - e^{-\tau(L)}\right) \rightarrow S$$

$T_B \rightarrow T_{source}$

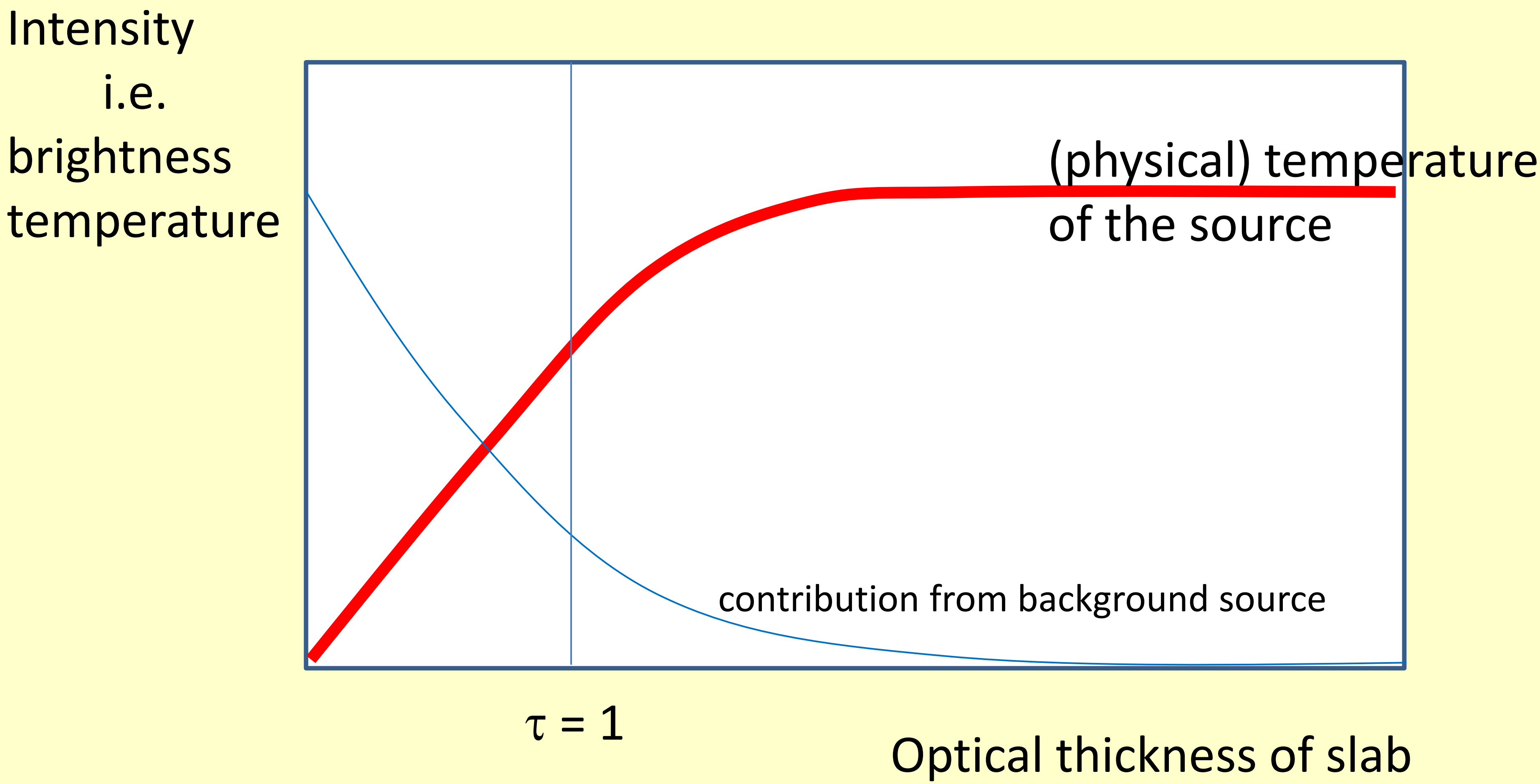
- ditto, **optically thin** limit:
  $\tau(L) << 1$ --> $\exp(-x) \approx 1-x$:

$$I \approx I_0\, e^{-\tau(L)} + j * L = I_0 + S * \tau(L)$$

$T_B \approx T_0 + T_{source} * \tau(L)$

measures the amount of emitting matter

Radiative Transfer (IV)



Physical processes that produce radio emission

- In principle: interaction of electromagnetic wave with charged particles

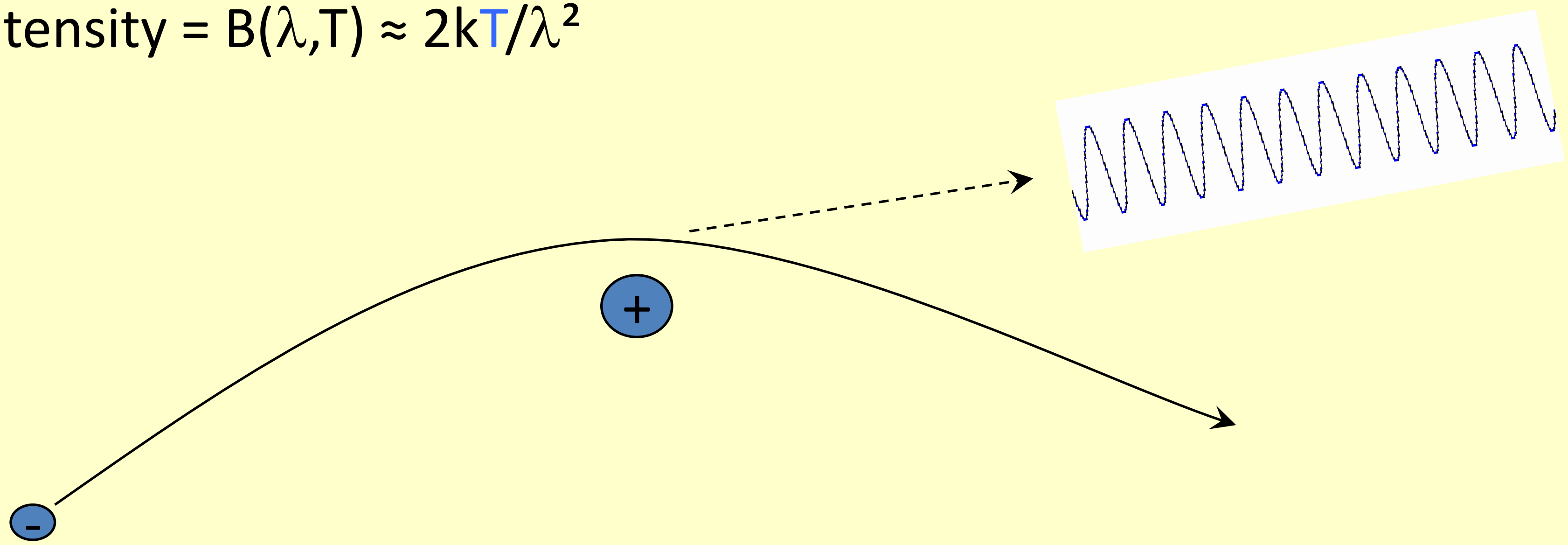
- **Continuum emission**: acceleration or deceleration (braking) of charged particles (electrons, protons)

- **Line emission**: downward transition of an electron between two bound states in an atom or molecule

Thermal emission (Bremsstrahlung):

by the random thermal motions of electrons passing by other charged particle (no polarization)

from opaque bodies: blackbody spectrum, at radio frequencies: intensity = $B(\lambda,T) \approx 2kT/\lambda^2$



Radiation by accelerated charges

Single charge *q* with acceleration *a*: power emitted into unit solid angle:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\, a^2\, \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$$

moving with speed *v* (β = *v/c*) parallel to acceleration, angle θ between line of sight and velocity vector

(Liénard 1898, v<<c: Larmor 1897)

Gas of thermal electrons

- Maxwell distribution of velocities ($\varepsilon = mv^2/2$)

$$n(\varepsilon) = \frac{2N}{\sqrt{\pi\,(kT)^3}}\,\sqrt{\varepsilon}\,e^{-\frac{\varepsilon}{kT}}$$

- Emissivity (cgs units: erg s^{-1} cm^{-3} Hz^{-1} $sterad^{-1}$)

$$j_f(f) = N_e N_i \frac{8}{3}\sqrt{\frac{2\pi\,m}{3\,kT}}\,\frac{Z^2 e^6}{m^2 c^3}\,g(f,T)\,e^{-\frac{hf}{kT}}$$

Gaunt factor *g(f,T)*

details: K.R.Lang, Astrophysical Formulae

The inverse process: absorption

- Kirchhoff's law (1860)

$$j_f = \kappa_f \; * \; B_f\,(f, T)$$

- ➔Absorption coefficient (cgs units: cm^{-1})

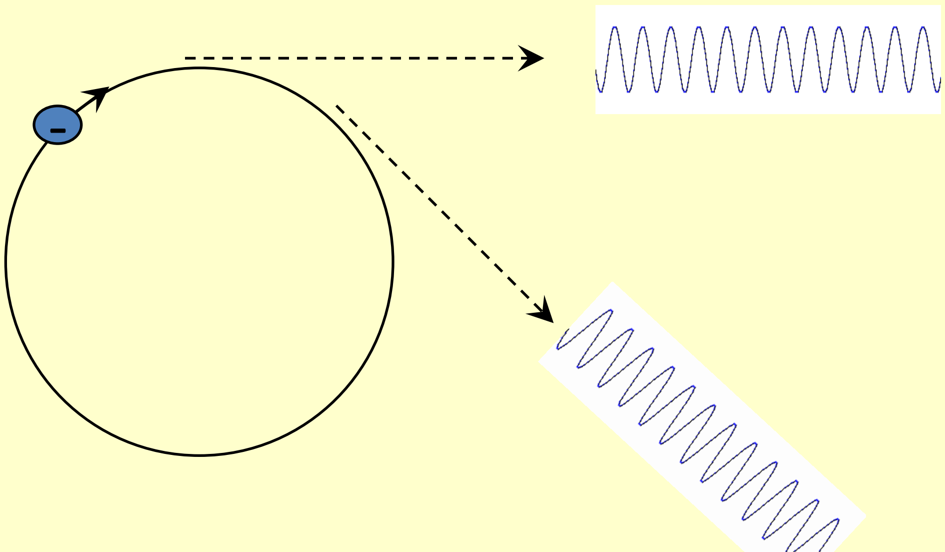
$$\kappa_f \approx \frac{0.009786\, N_e\, N_i}{f^2 T^{\frac{3}{2}}} \ln(4.954\; 10^7 \left(\frac{T^{3/2}}{Zf}\right)) \qquad \text{T < 316000 K}$$

$$\kappa_f \approx \frac{0.009786\, N_e\, N_i}{f^2 T^{\frac{3}{2}}} \ln(4.954\; 10^{10}\, T/f) \qquad \text{T > 316000 K}$$

Note: absorption increases with wavelength

Synchrotron emission

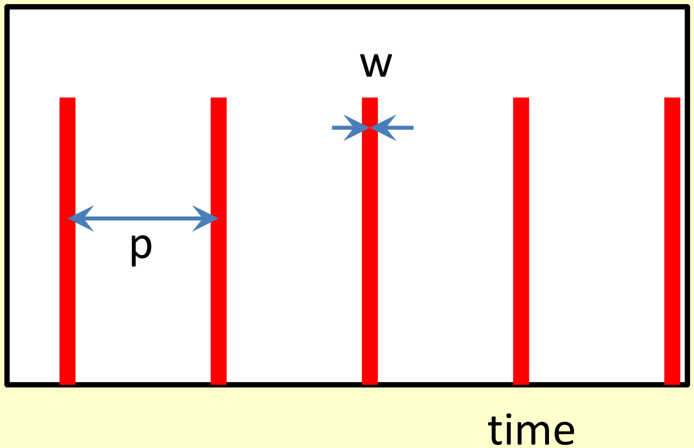
Deflection of electrons in a magnetic field
➔ electrons spiral around the magnetic field lines and emit a pulse whenever their motion is directed towards us
➔ superposition of contributions by all electrons in the plasma ((thermal) distribution of speeds) gives:
continuous spectrum (decreases with frequency, high frequency cutoff, linear polarization)

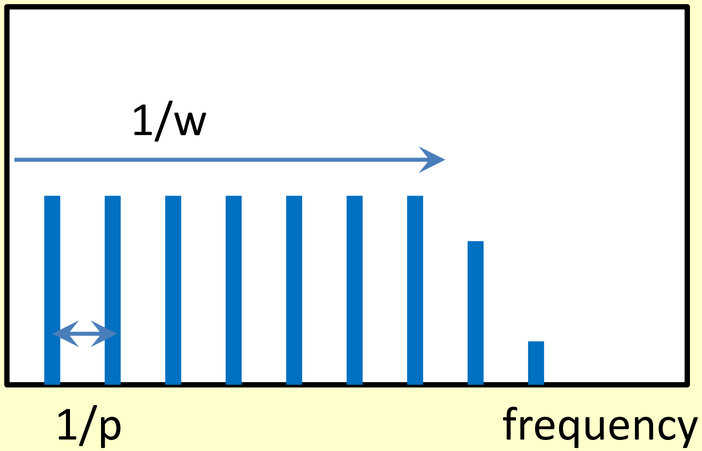


Synchrotron emission

At high speeds the distribution becomes a narrow cone with half width $\theta = \gamma = \sqrt{1-\beta^2}$ (from Liénard's equation, see above)

Electron in circular orbit of radius ρ with period p = v/ρ ➔ pulse width w = γ^3/p ➔ observer sees harmonics of the rotation frequency 1/p, up to the maximum number γ^3





Synchrotron emission

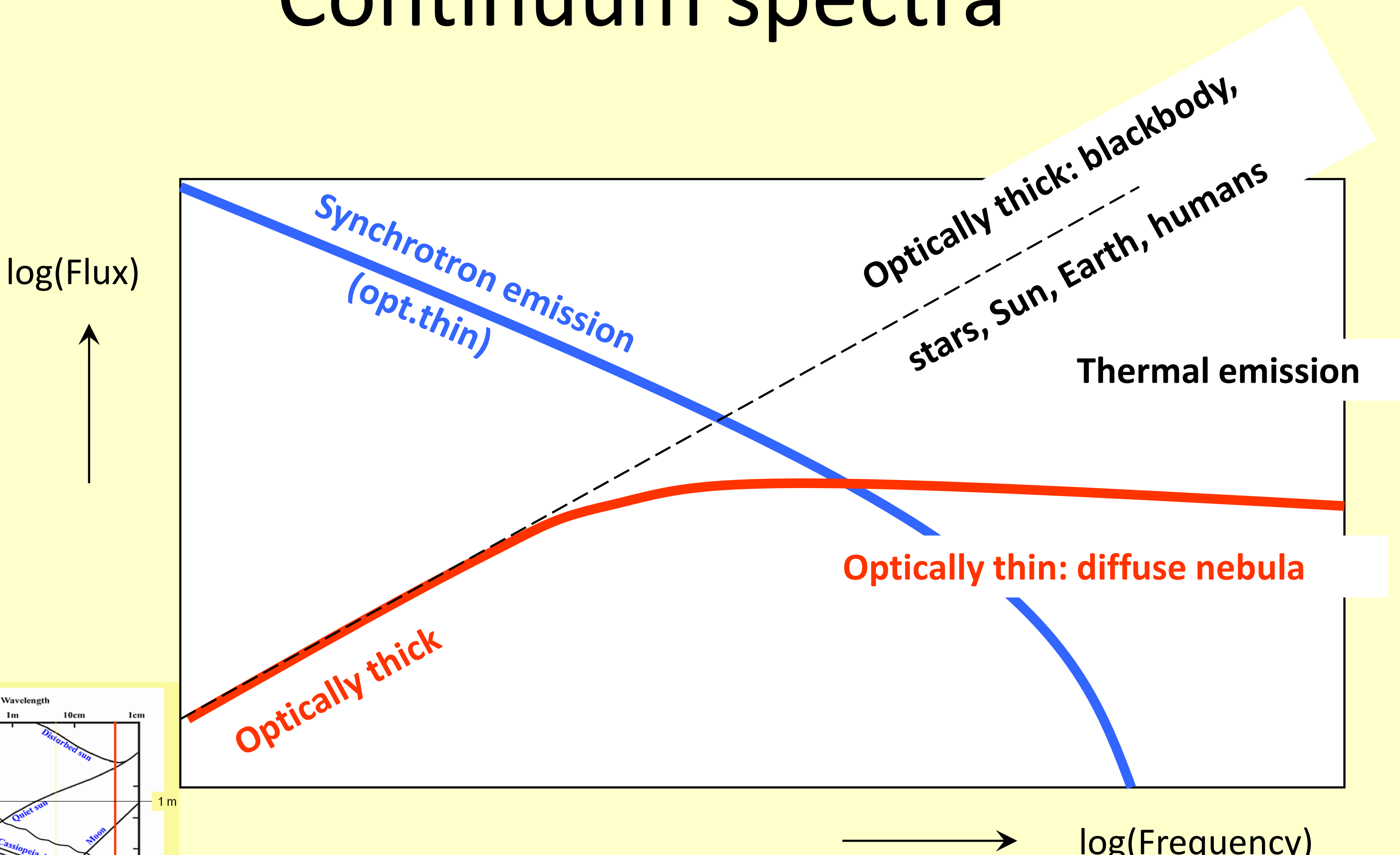
If one considers a population of electrons, distributed in energies like a power law: $dN/dE = cte * E^x$

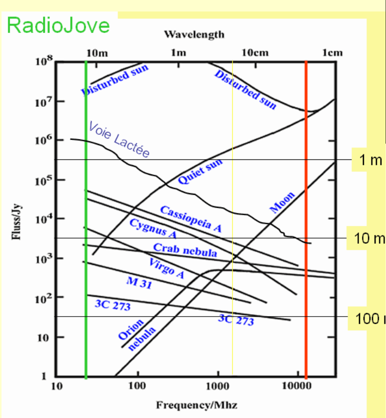
it can be shown … ☺ …

that the frequency spectrum of the emitted radiation follows a power law with exponent $-(x-1)/2$

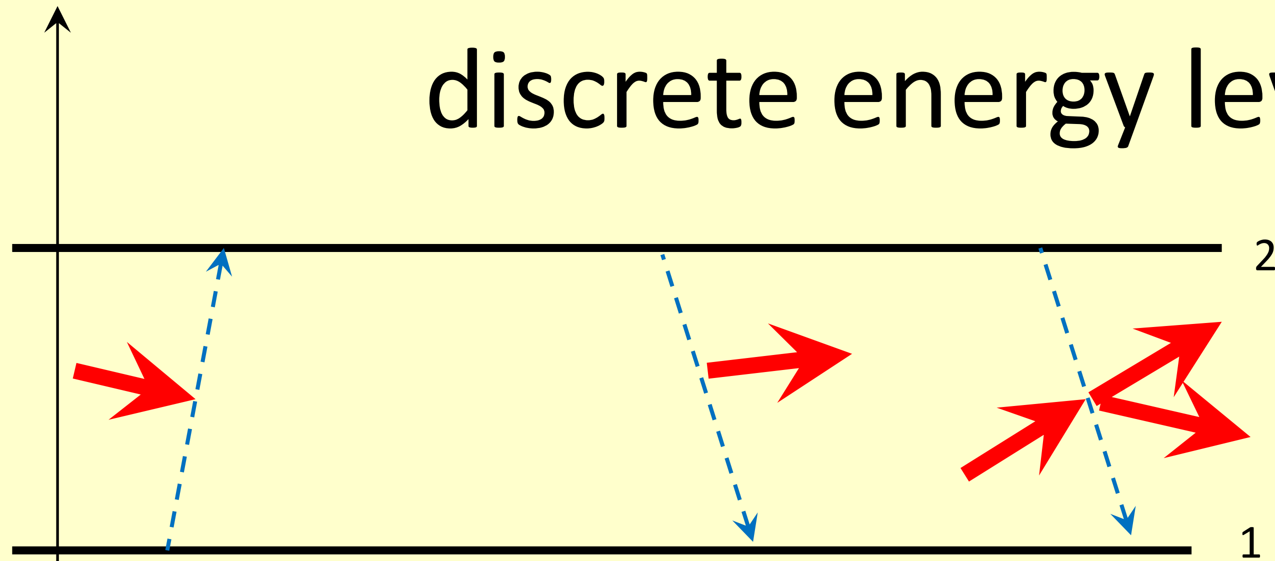
details: K.R.Lang, Astrophysical Formulae

Continuum spectra





Lines = transitions between discrete energy levels



Absorption	Spontaneous Emission	Stimulated Emission
rate = $n_1 B_{12} * J_{12}$	rate = $n_2 A_{21}$	rate = $n_2 B_{21} * J_{12}$

Einstein coefficients:

$g_1 B_{12} = g_2 B_{21}$ g_i = statistical weight of level i

$A_{21} = B_{21} * 2hf^3/c^2$ = transition probability

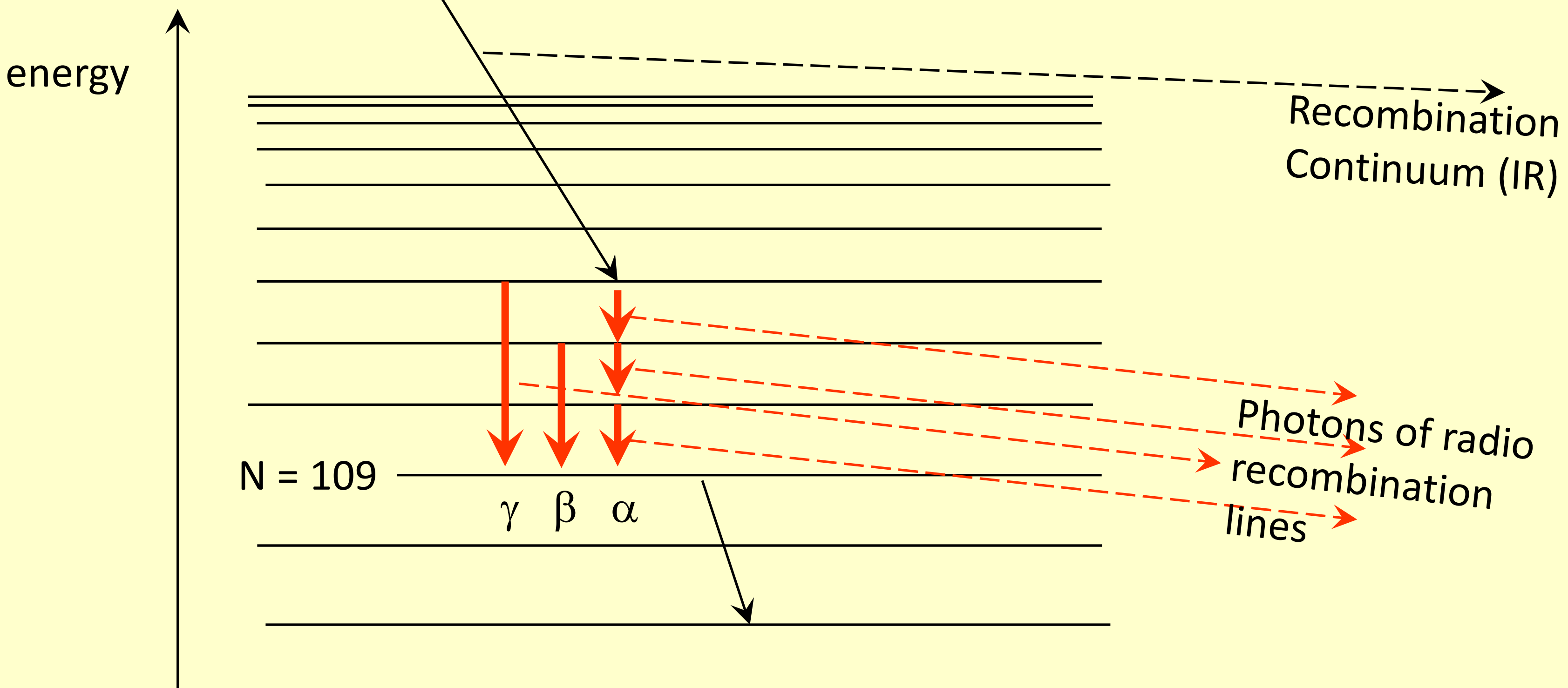
= inverse lifetime of upper state

$\kappa_{12} = hf_{12}/4\pi * (n_1 B_{12} - n_2 B_{21}) * \varphi(f)$ absorp.coeff. with line profile function φ

$j_{21} = hf_{12}/4\pi * n_2 A_{21} * \varphi(f)$ emissivity

Line emission (I)

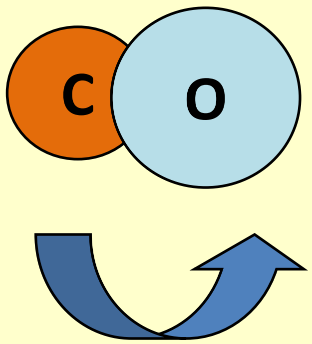
- Electron in hydrogen atom flips its spin (21 cm line at 1420.406 … MHz, A = 2.85 10^{-15} s^{-1} (magn.dipole))
- Electron changes energy state in atom (mostly hydrogen after recombination of proton and electron in plasma: recombination lines, H109α 5 GHz, A $\approx$ 10^{8} s^{-1} (electr.dipole))

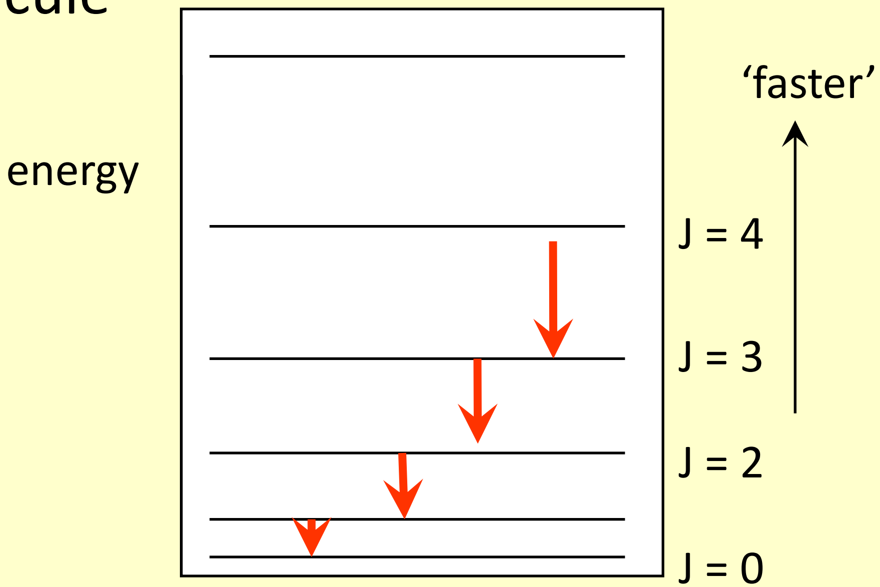


Line emission (II)

Molecule slows down from its high rotational state (into which it had been excited e.g. by a collision with another molecule (H_2))

CO = polar molecule

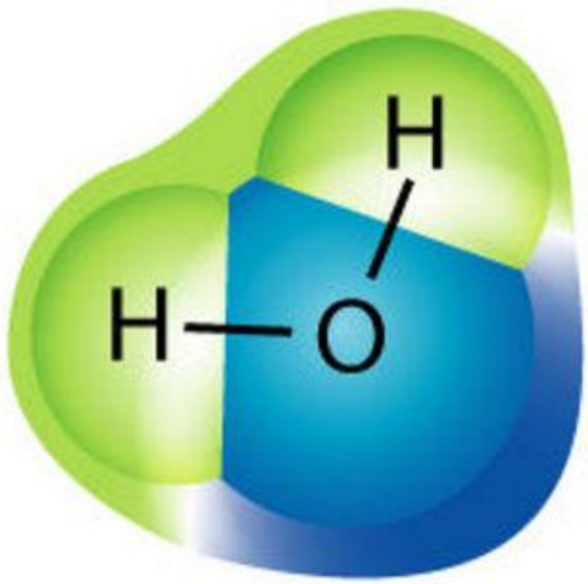


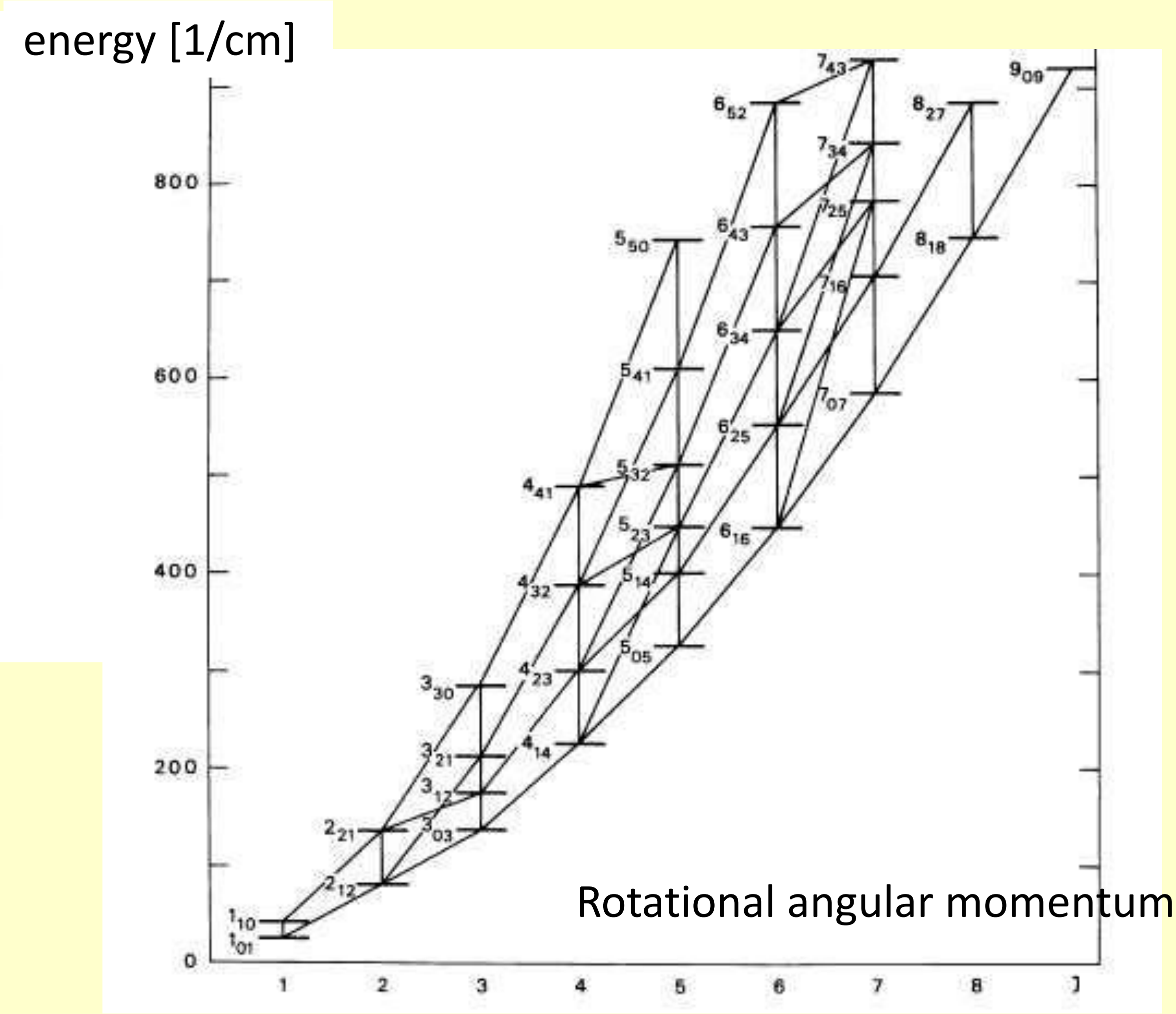


Examples: CO lines at 110, 220, 330, … GHz $A \approx 10^{-7}\ s^{-1}$

Line emission (III)

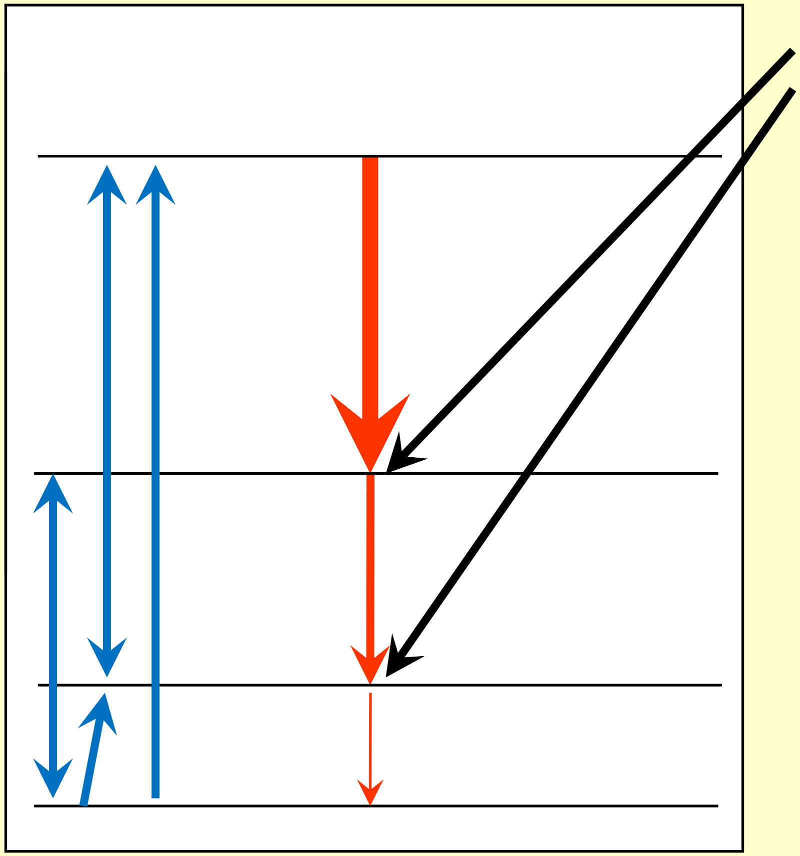
Water (asymmetric polar molecule) 22 GHz





Line emission (IV): Masers

energy



collisions

'traffic jam' leads to overpopulation of upper levels $n_2 > n_1$

➔ **negative** absorption coefficient

$$\kappa_{12} = hf_{12}/4\pi * (n_1 B_{12} - n_2 B_{21}) * \varphi(f)$$

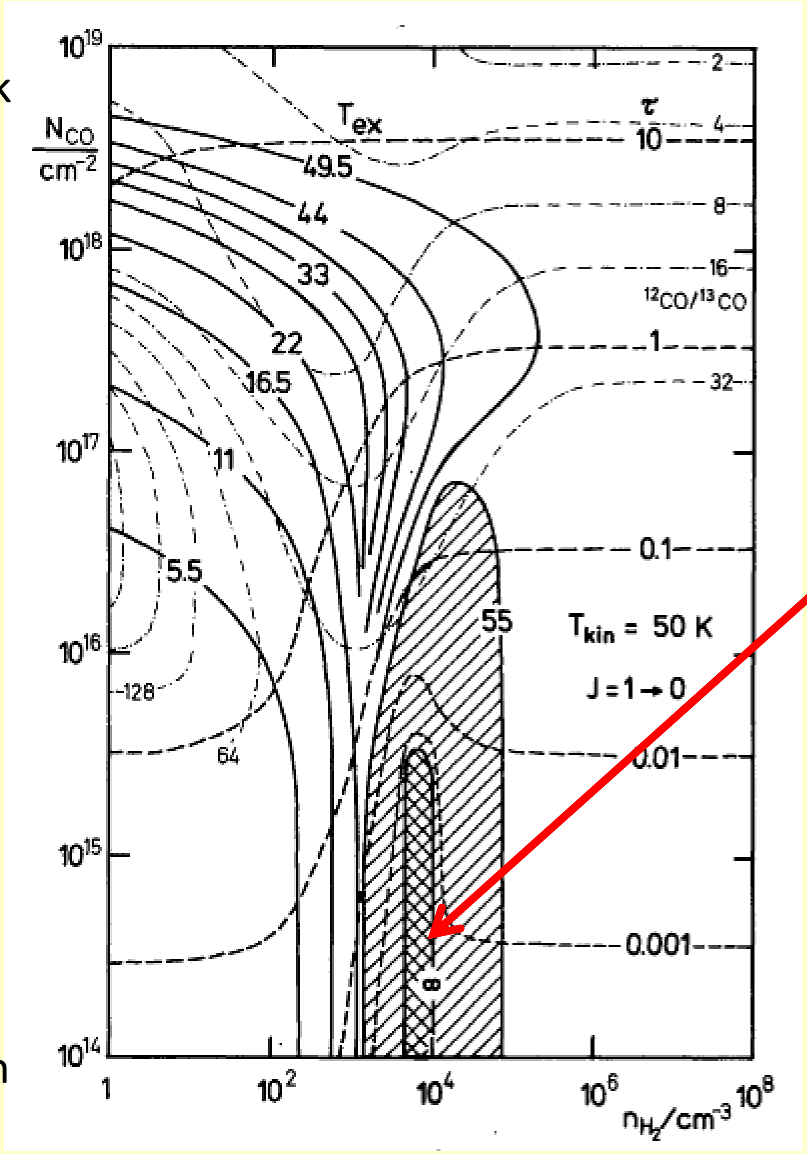
➔ Incoming radiation is amplified

Examples:

- OH 1.7 GHz
- H_20 22 GHz
- SiO 43 GHz
- CH_3OH 6.6 and 12 GHz

Line emission (V): Masers

Optically thick



Optically thin

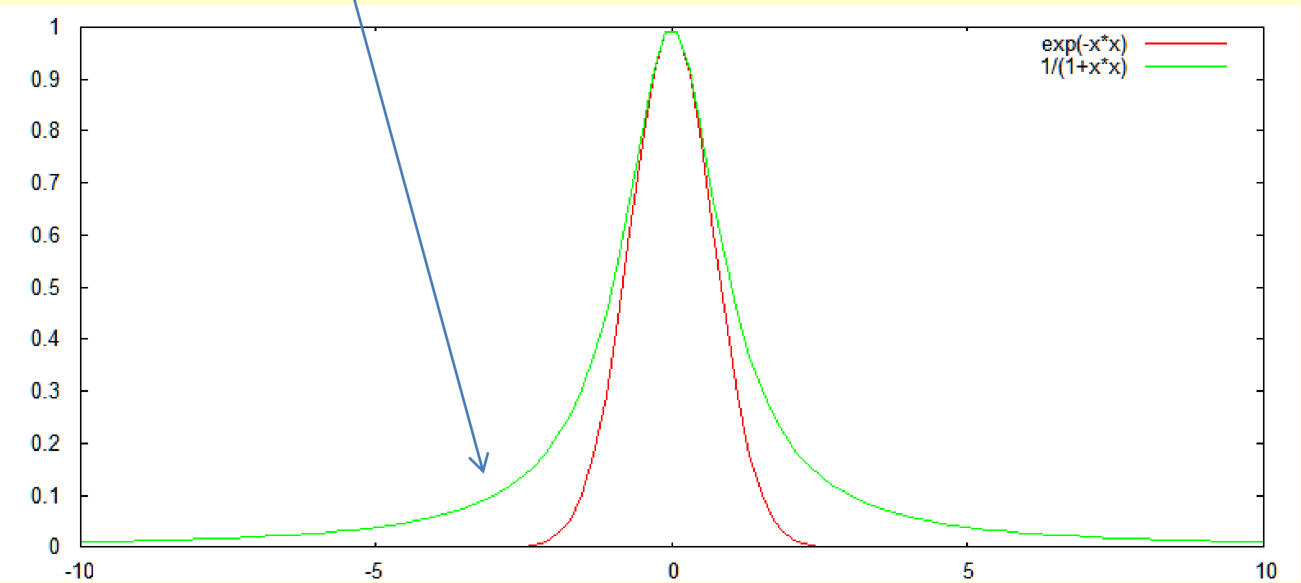
(weak) maser effects can also occur with CO under certain conditions of Temperature, density, cloud column density

Köppen & Kegel (1980) A&AS 42, 59

Line emission (VI)

The **width** of a line is determined by

- Lifetime of transition: 1/A
  - HI 21 cm line: $\Delta f/f = 2.85\ 10^{-15}/1.4\ 10^{9} \approx 10^{-24}$!!!
  - Line profile is Lorentzian shape ($\varphi \propto 1/(1+(\Delta f)^2)$)



Line emission (VI)

- Thermal and small-scale turbulent motions (Doppler effect) of the emitting gas
  - **Gaussian** profile: width $\Delta f_D = f/c * \sqrt{\frac{kT}{\mu} + \xi^2}$
  - hydrogen atoms at T=100 K give v_D = 1km/s
  - HI 21 cm line: 1 km/s ➔ Δf_D = 4.7 kHz
  - Motions of HI clouds in the ISM: 10 km/s
  - ➔ shape of HI 21 cm features reveals the kinematics and dynamics of the HI gas

Interpretation of Lines (I)

With Boltzmann formula define from level population ratio an **Excitation Temperature** (for this transition):

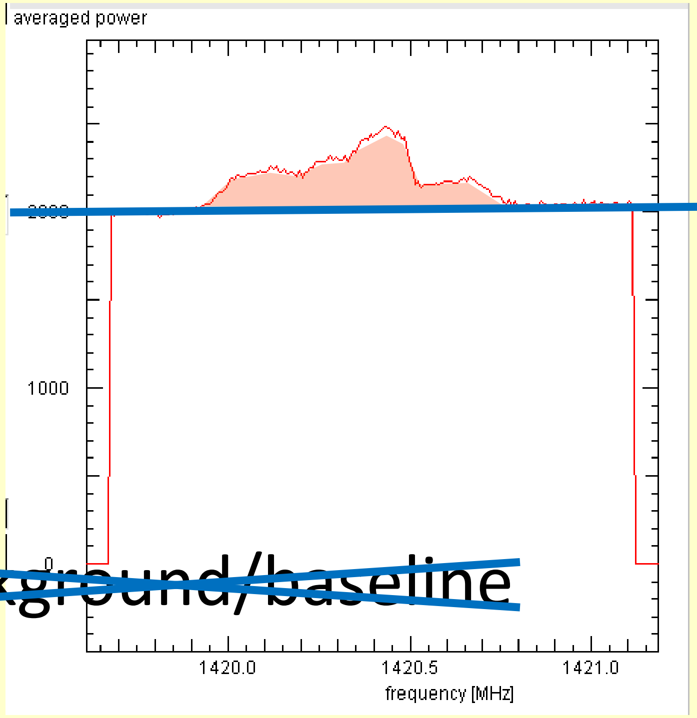
$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp\left(-\frac{hf}{k\boldsymbol{T_{ex}}}\right)$$

The ratio of emissivity and absorption is equal to the Planck function of this temperature:

$$S = \frac{j}{\kappa} = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} = \frac{2hf^3}{c^2} \frac{1}{\frac{n_1 g_2}{n_2 g_1} - 1}$$

$$\boxed{S = B(f, T_{ex})}$$

Interpretation of Lines (II)



Optically thin line radiation lies 'on top ' of any continuum or background:

- In terms of velocity shift:

$$I(v) = \tau(v) * B(T_{ex})$$ ~~+ background/baseline~~

- Integrate this emission excess over the entire line profile gives the total line emission

$$\int I(v) dv = B(T_{ex}) \int \tau(v) dv$$

Interpretation of Lines (III)

- Integrate optical depth over line profile

$$\int \tau(v)dv = \iint \kappa(v,s)dsdv$$

$$= \frac{hf}{4\pi}\int (n_1 B_{12} - n_2\, B_{21})ds \int \varphi(v)dv \quad (= 1)$$

$$= \frac{hf}{4\pi} B_{12}(1 - e^{-hf/kT_{ex}}) \int n_1 ds$$

- **Column density** of matter along the line of sight: (N.B. $n=n_1+n_2$)

$$\mathrm{N} = \int n\, ds$$

Interpretation of Lines (III)

All together (with observed brightness temperatures):

$$\int T_B(v)dv = cte \, * N \, * \left(1 - e^{-hf/kT_{ex}}\right) T_{ex}$$

Thus, from the 21cm line one gets HI column density:

$$N_H = 1.822\ 10^{18}\ \frac{1}{T_{ex}} \int T_B dv$$

in practical units: [atoms/cm^2] column density, [K] temperatures,
[km/s] velocity, for 21 cm line *Tex* is called 'spin temperature'

Interpretation of Lines (IV)

Caveats:

- To get column density, one needs the **brightness** temperature
- This is equal to the measured antenna temperature only if the source is **well resolved**!