Introduction to Radioastronomy:

Interferometers and Aperture Synthesis

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http://astro.u-strasbg.fr/~koppen/JKHome.html

Problem No.2: Angular resolution

- Diffraction limit: to distinguish two point objects with an instrument of aperture diametre D at wavelength λ, they must be separated by an angle larger than

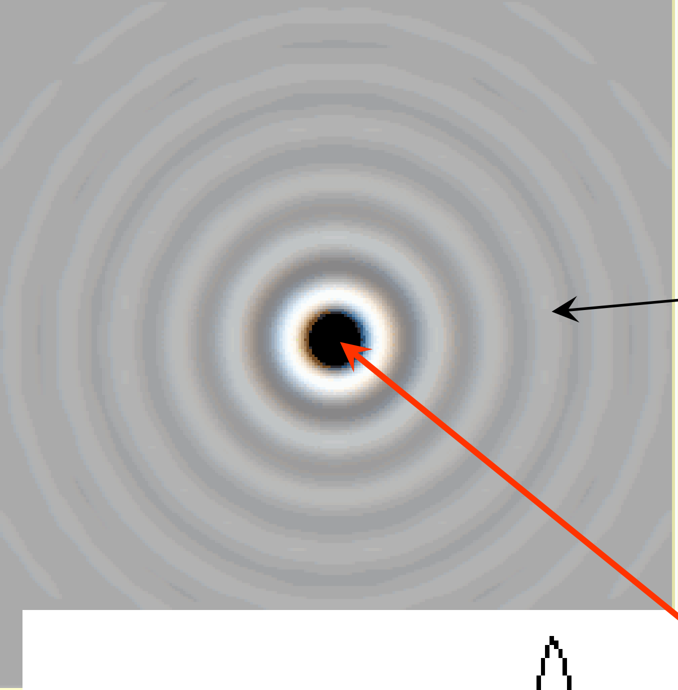
$$\sin \alpha > \lambda/D$$

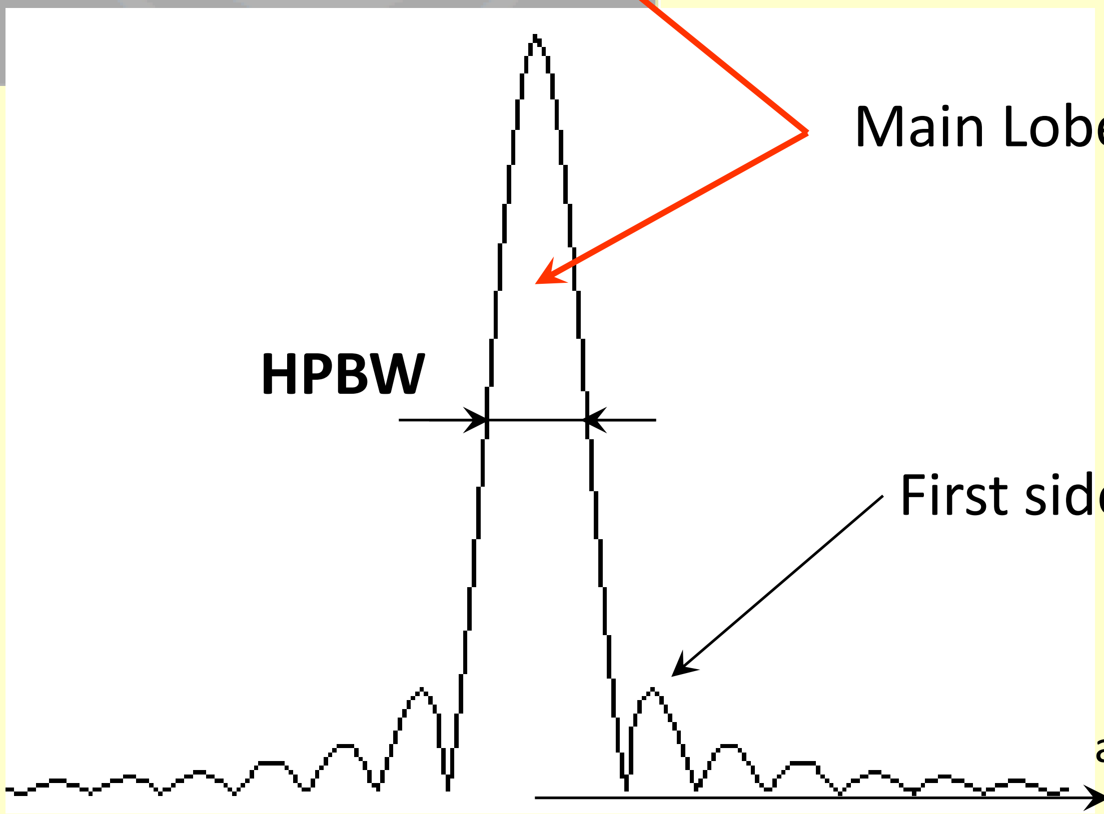
	diametre	wavelength	resolution
Human eye	2 mm	500 nm	50 arcsec
ESA-Dresden	120 cm	3 cm	1.5 deg
Arecibo	300 m	21 cm	2 arcmin
Effelsberg	100 m	3 cm	1 arcmin

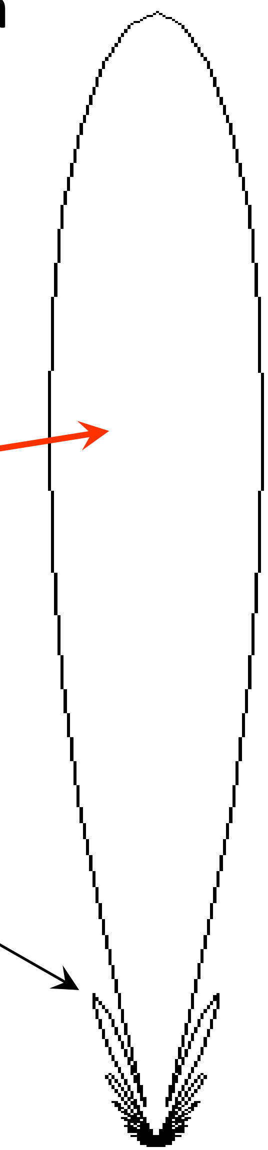
Antenna pattern

= its angular sensitivity curve

= is the interference pattern of its aperture



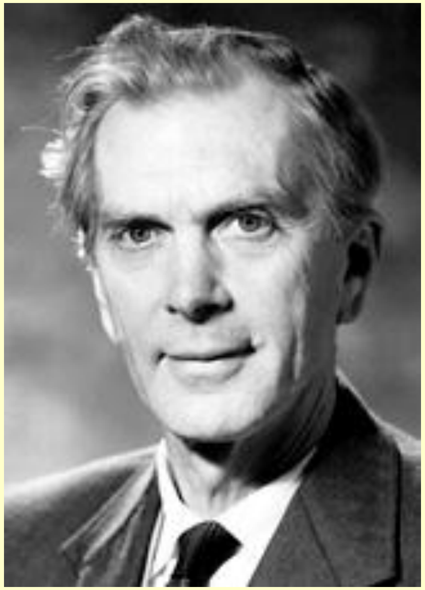




Interferometry/Aperture Synthesis

- Combining the outputs of several radio telescopes placed some distance **B** (baseline) gives the same angular resolution of an instrument of that size

- 1946 M.Ryle (Cambridge, U.K.)

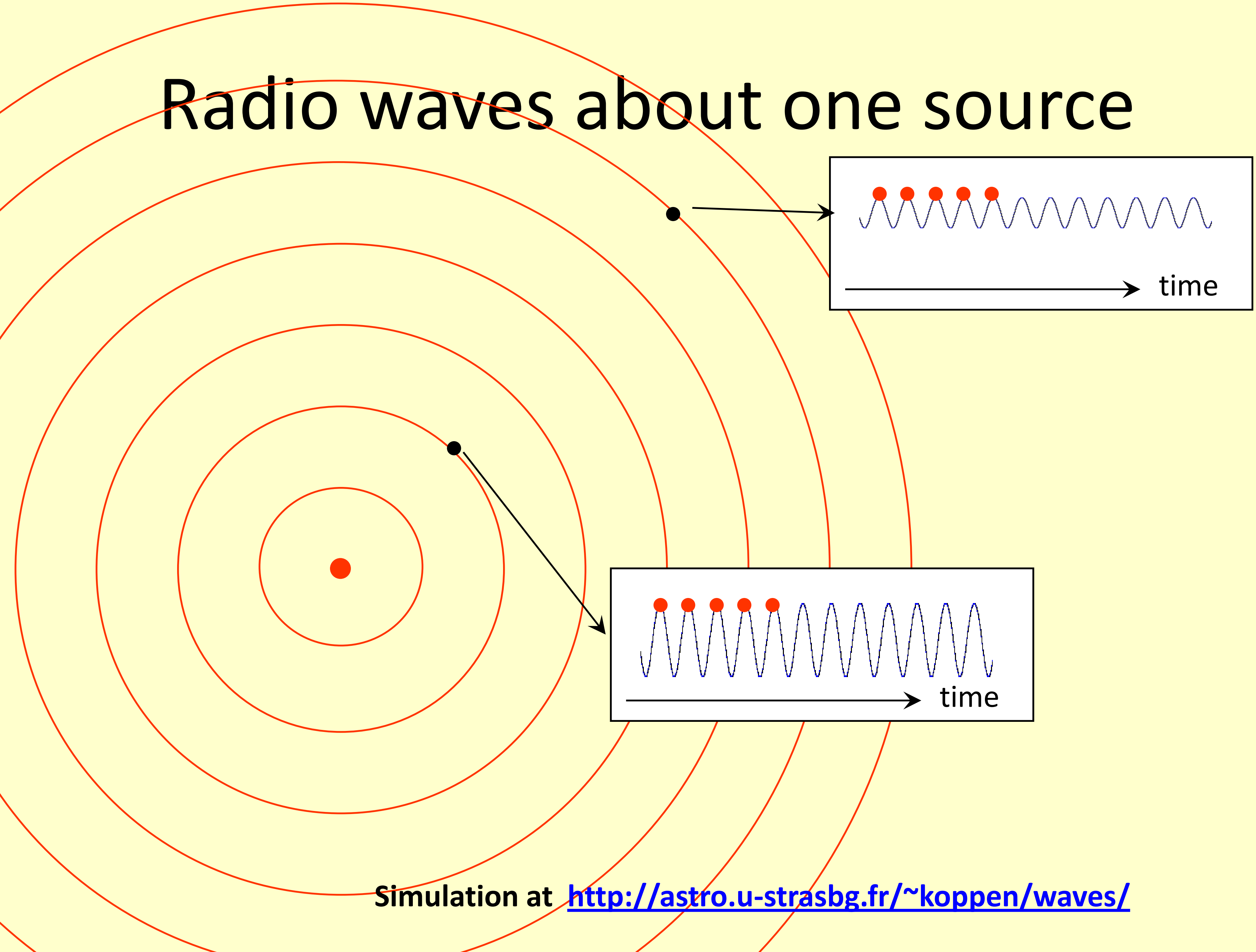


Interference: a word with double meaning

- (technical sense) = any signal or noise which is also picked up, and which messes up reception or observations

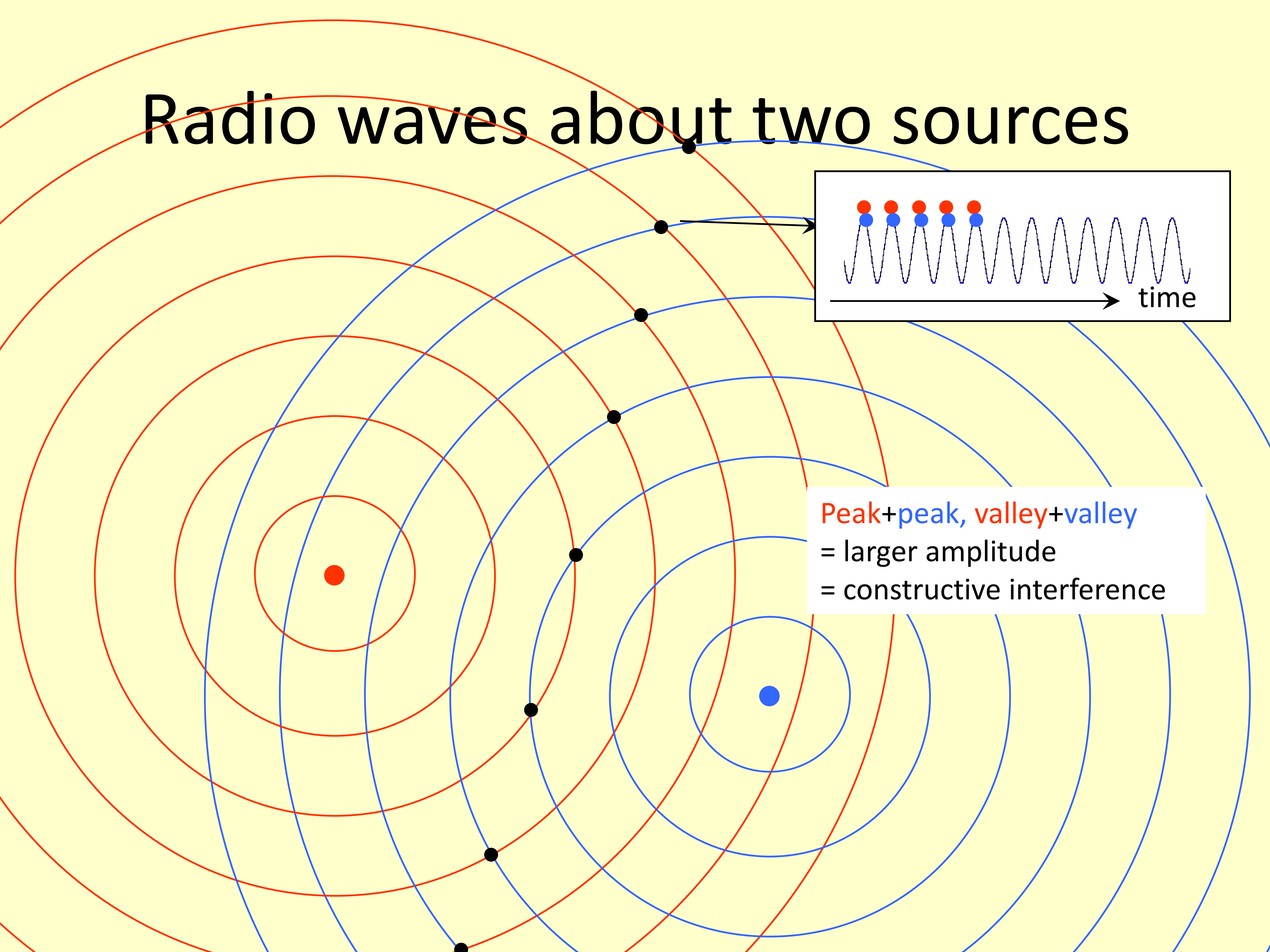
- (physical sense) = the result of the superposition of waves (of any type)

Radio waves about one source

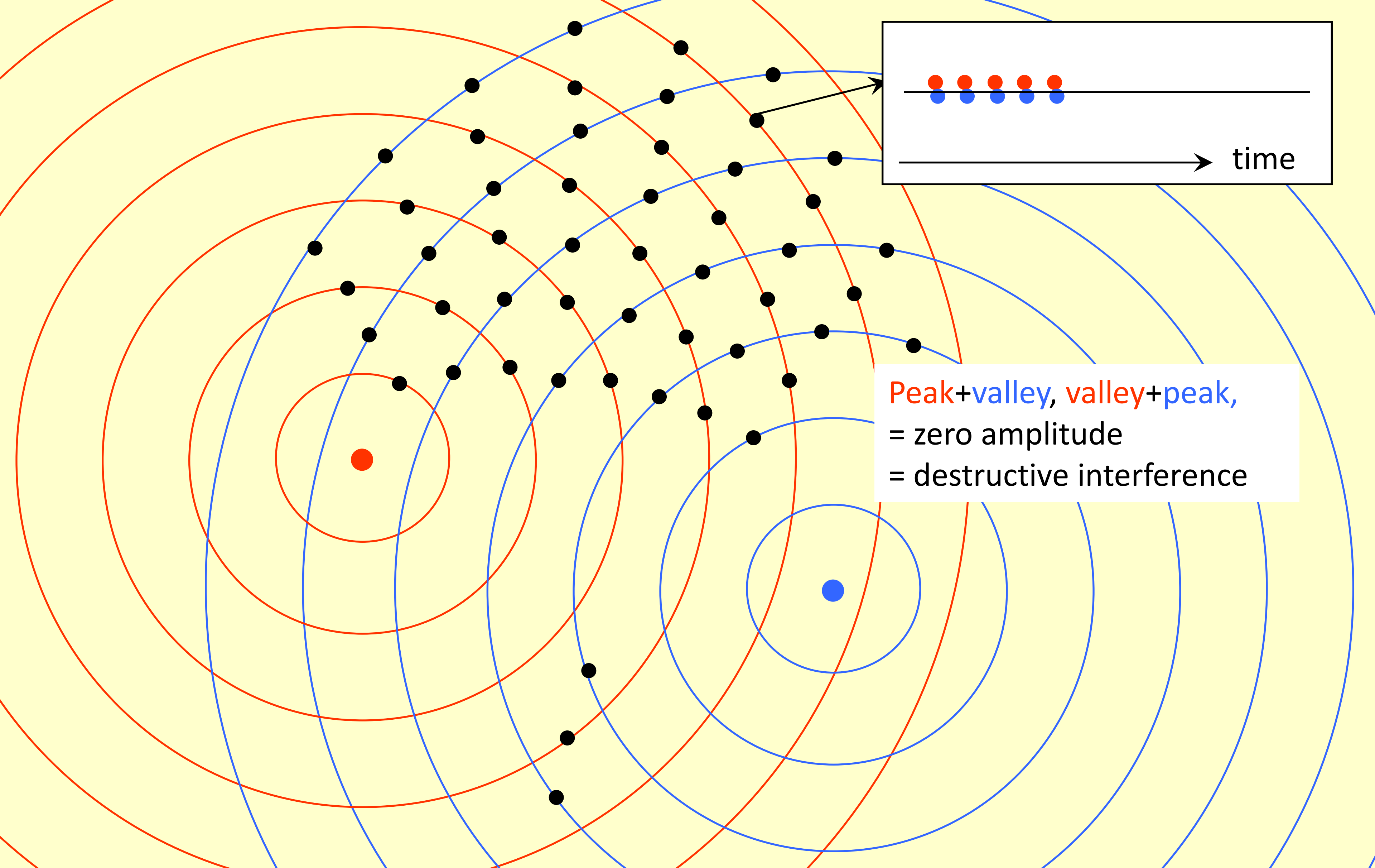


Simulation at http://astro.u-strasbg.fr/~koppen/waves/

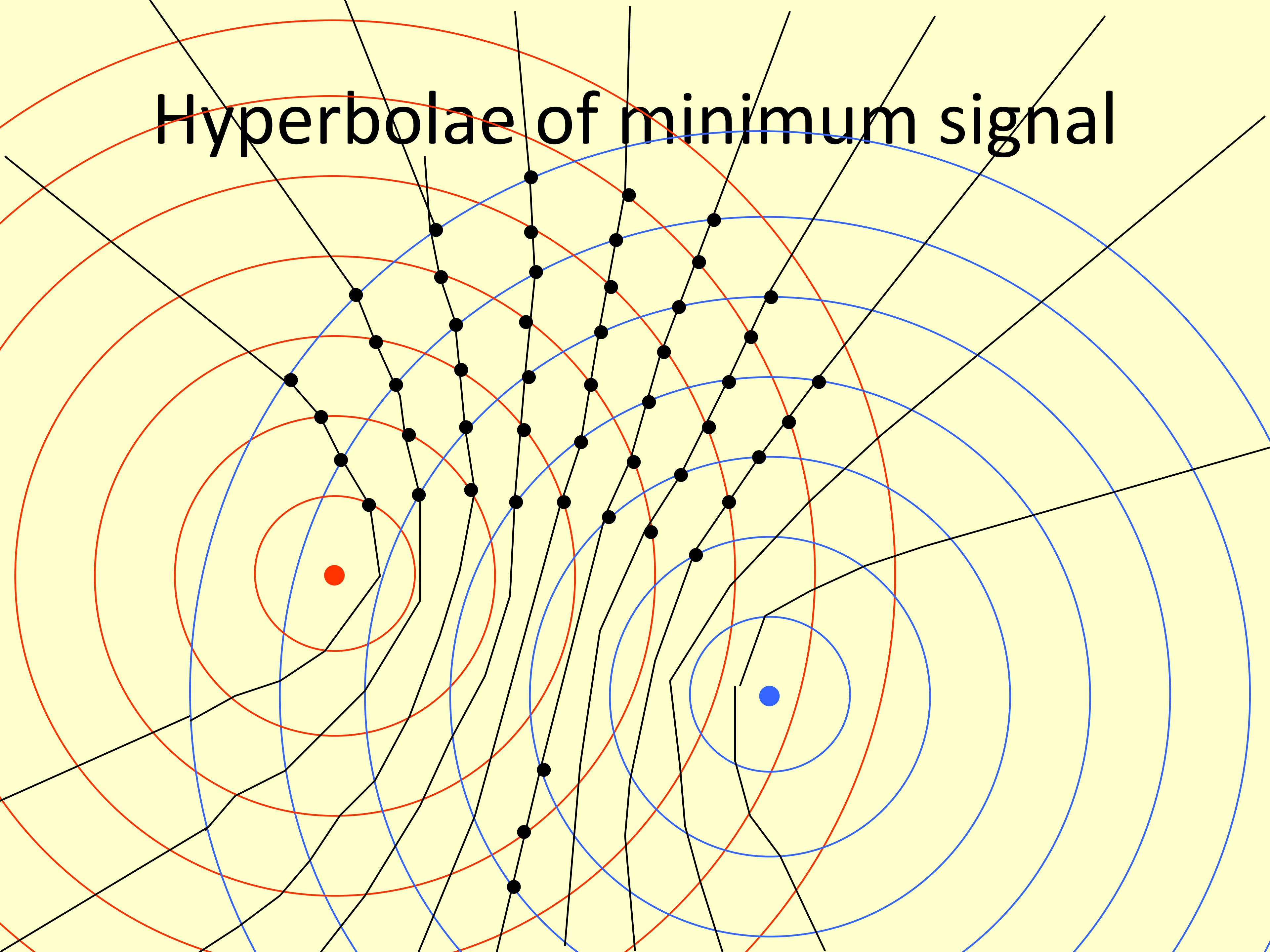
Radio waves about two sources



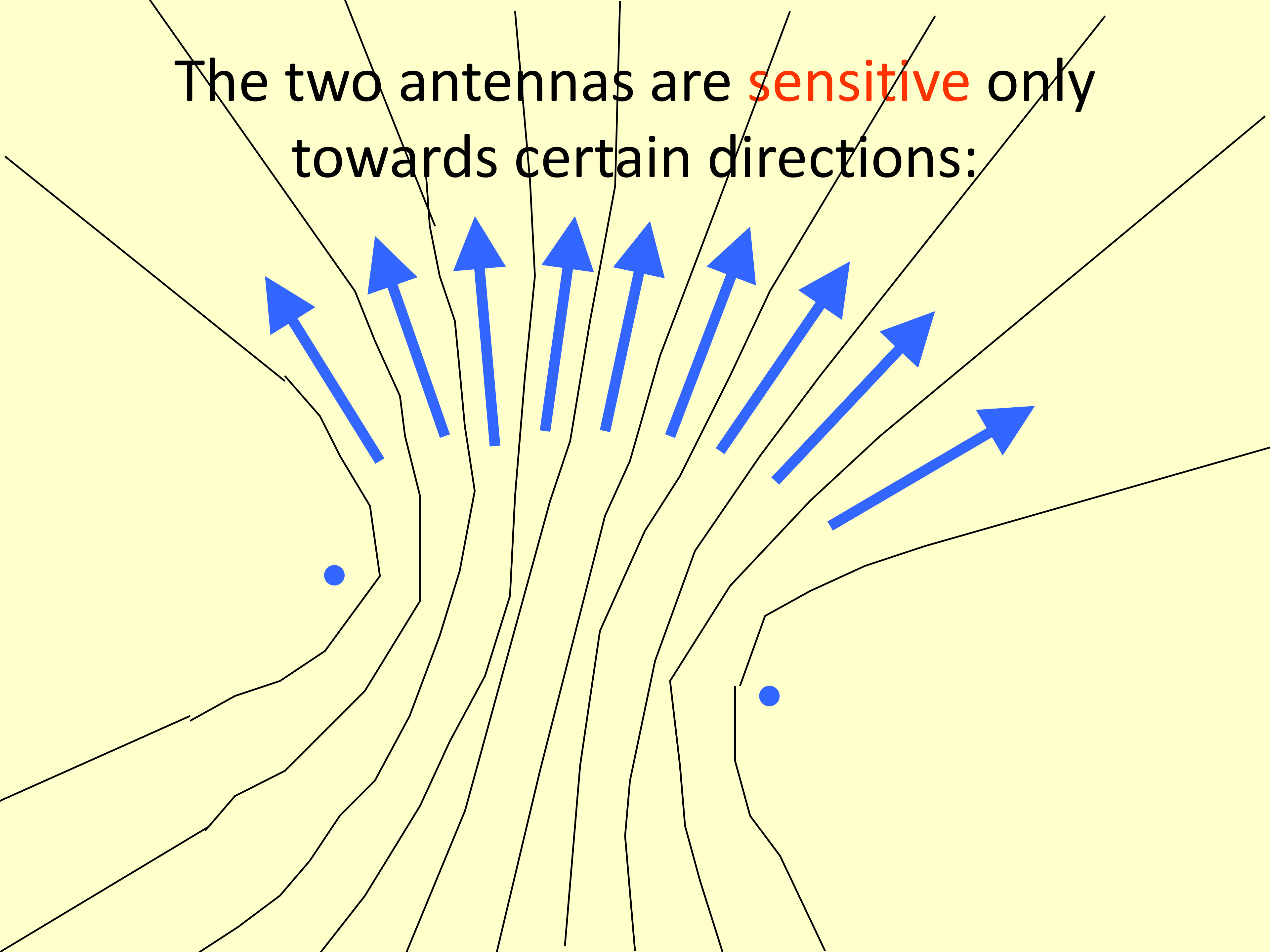
Radio waves about two sources



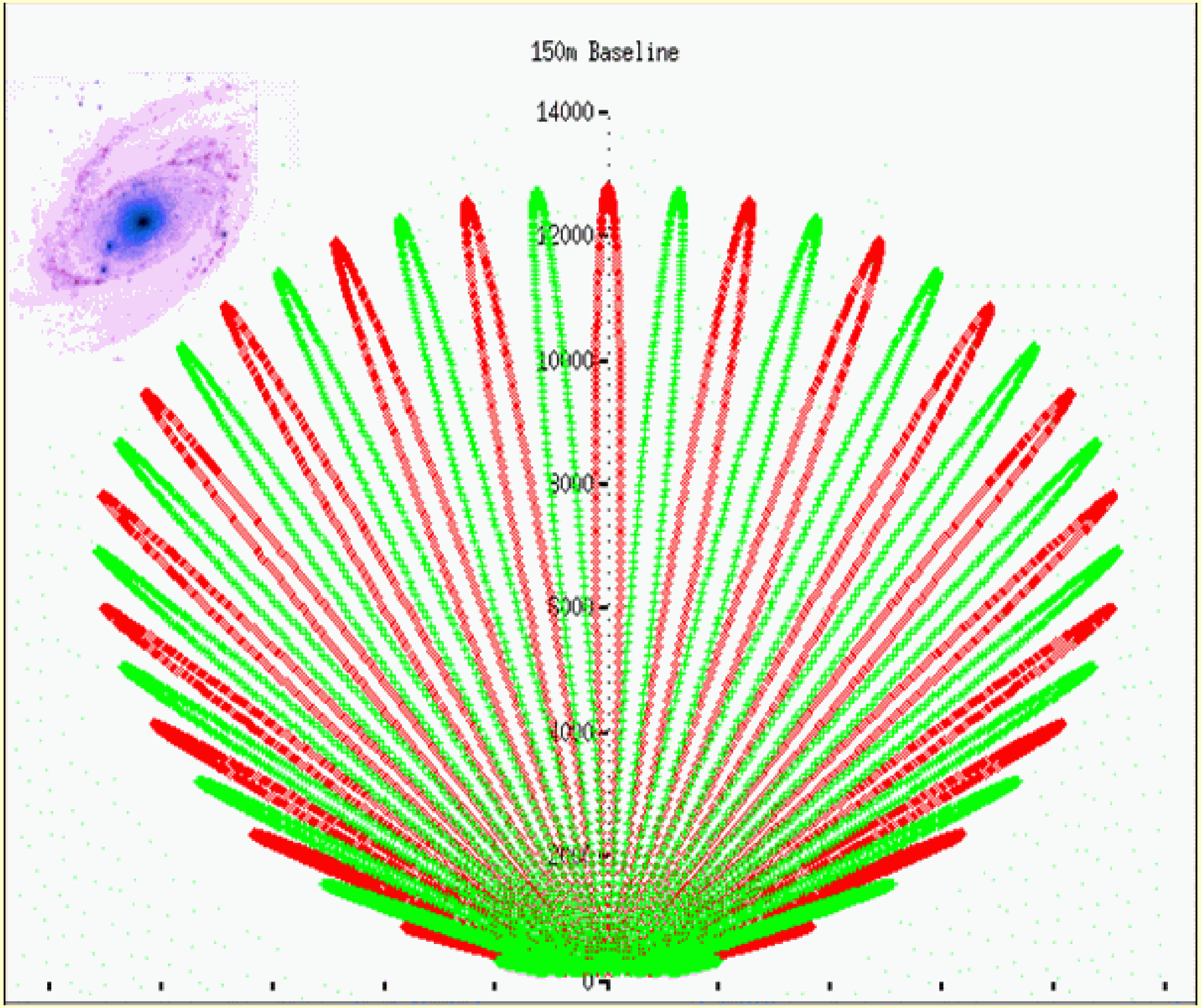
Hyperbolae of minimum signal



The two antennas are sensitive only towards certain directions:



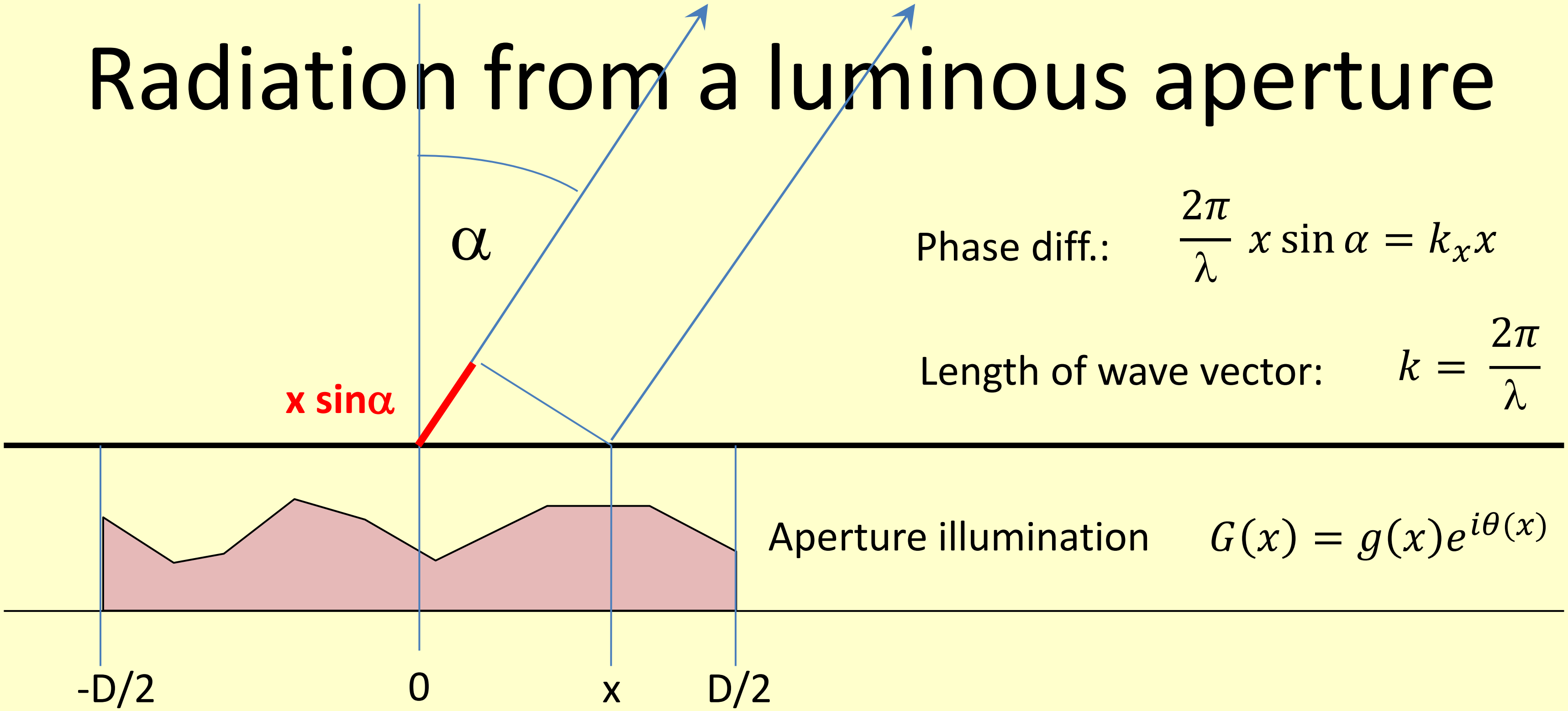
The antenna pattern



Reciprocity

The antenna pattern at reception is identical to the pattern at transmission

Radiation from a luminous aperture



Phase diff.: $\frac{2\pi}{\lambda}\, x \sin\alpha = k_x x$

Length of wave vector: $k = \frac{2\pi}{\lambda}$

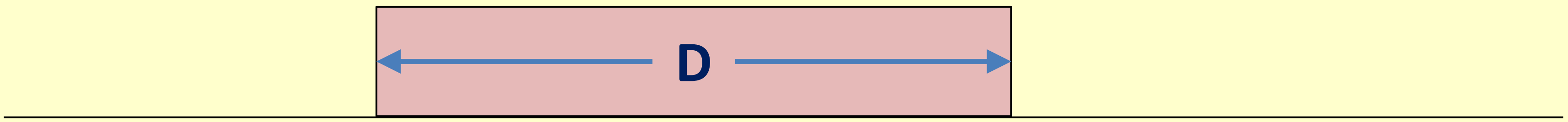
Aperture illumination $G(x) = g(x)e^{i\theta(x)}$

The amplitude of the **electric field** (at large distance)
is the sum of contributions from all parts of the aperture:

$$E(\alpha) = \int G(x)\, e^{ik_x x} dx = \int g(x) e^{i(\theta(x)+k_x x)} dx$$

… nothing but the **Fourier transformation of the aperture illumination function**.

Case 1: uniformly illuminated dish



$$\mathrm{G}(x) = \frac{1}{D} \quad \text{for} \quad -\frac{D}{2} < x < \frac{D}{2} \quad ; = 0 \text{ everywhere else}$$

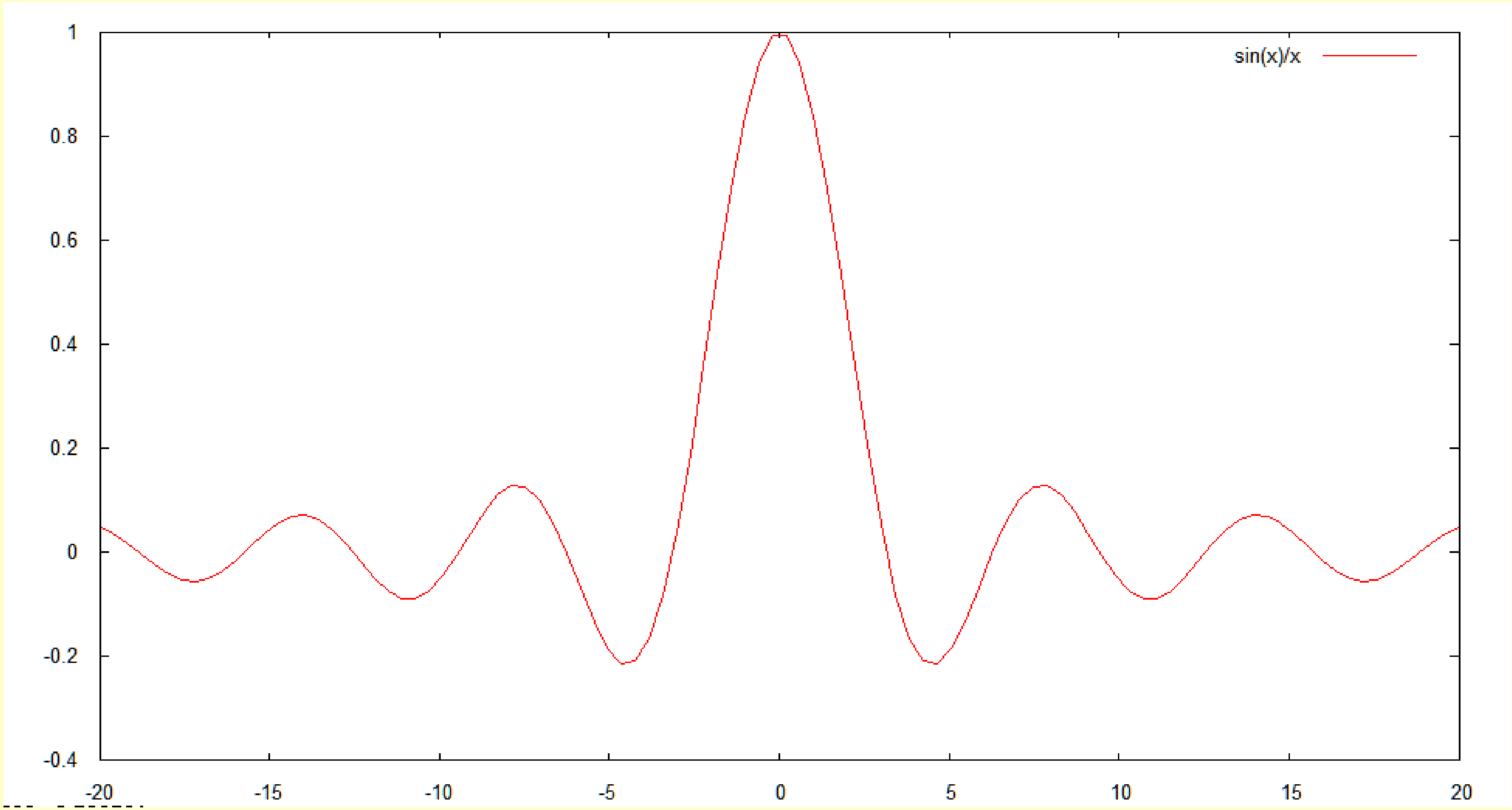
$$E(\alpha) = \int G(x)\, e^{ik_x x} dx$$

$$= \frac{1}{D} \int_{-D/2}^{D/2} e^{ik_x x} dx = \frac{e^{ik_x \frac{D}{2}} - e^{-ik_x \frac{D}{2}}}{ik_x D}$$

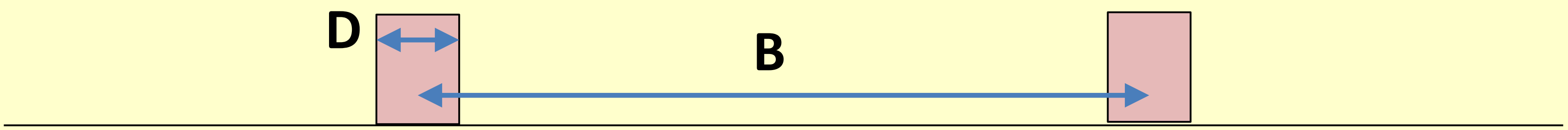
$$= \frac{\sin(k_x D/2)}{k_x D/2}$$ since $e^{ix} = \cos x + i \sin x$

$$= \operatorname{sinc}(k_x D/2)$$ the Fourier transform of a square pulse

Antenna pattern of single uniformly illuminated dish



Case 2: two-dish interferometer



$\mathrm{G}(x) = \frac{1}{D}$ for $-\frac{B}{2} - \frac{D}{2} < x < -\frac{B}{2} + \frac{D}{2}$ and $\frac{B}{2} - \frac{D}{2} < x < \frac{B}{2} + \frac{D}{2}$

$$E(\alpha) = \frac{1}{D}\int_{-B/2-D/2}^{-B/2+D/2} e^{ik_x x} dx + \frac{1}{D}\int_{B/2-D/2}^{B/2+D/2} e^{ik_x x} dx$$

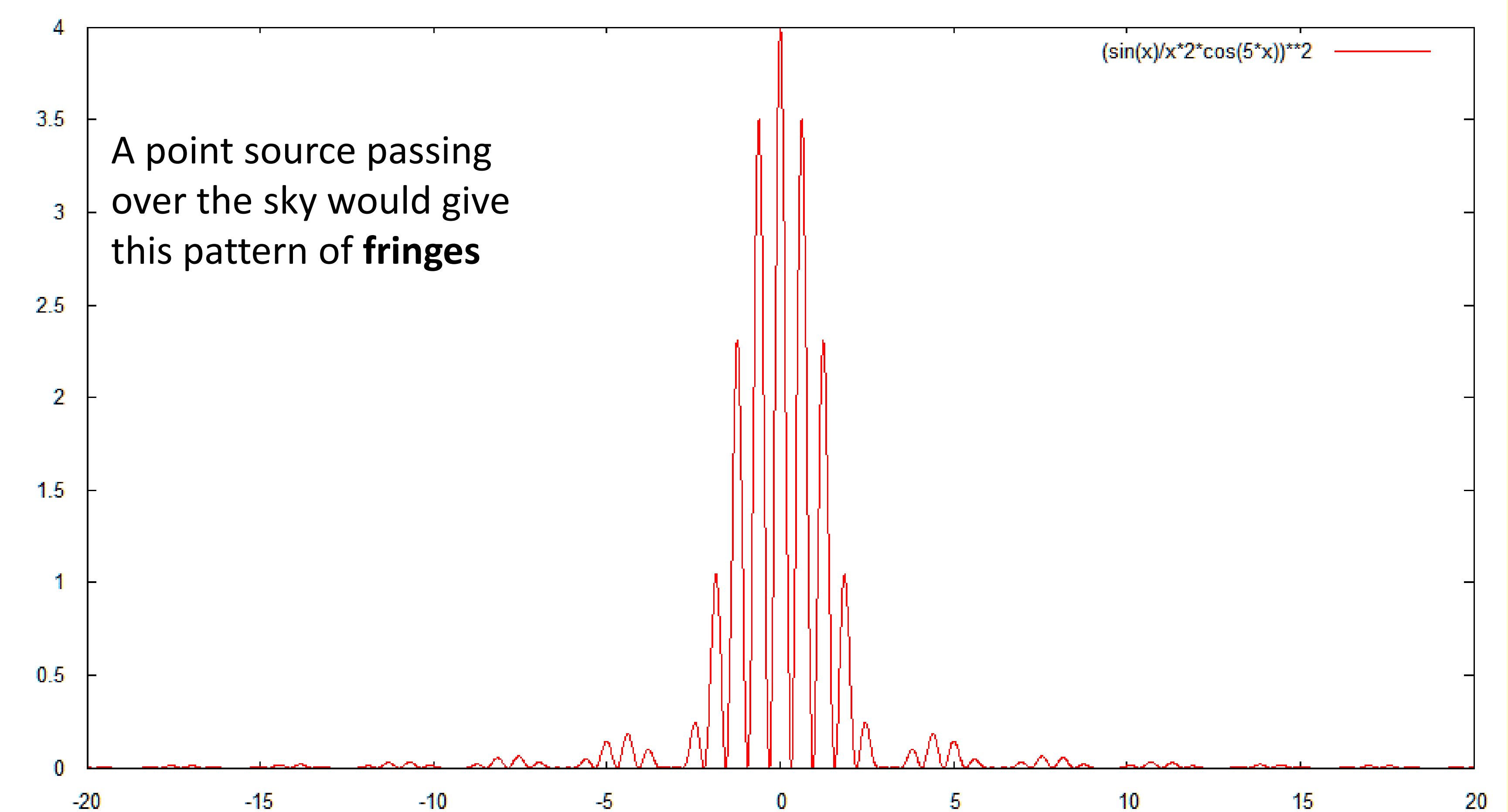
$$= \frac{e^{ik_x(-\frac{B}{2}+\frac{D}{2})} - e^{ik_x(-\frac{B}{2}-\frac{D}{2})}}{ik_x D} + \frac{e^{ik_x(\frac{B}{2}+\frac{D}{2})} - e^{ik_x(\frac{B}{2}-\frac{D}{2})}}{ik_x D}$$

$$= (e^{ik_x\frac{B}{2}} + e^{-ik_x\frac{B}{2}}) \frac{e^{ik_x\frac{D}{2}} - e^{-ik_x\frac{D}{2}}}{ik_x D}$$

$$= \cos(k_x \frac{B}{2}) \quad * \quad \mathrm{sinc}(k_x \frac{D}{2})$$

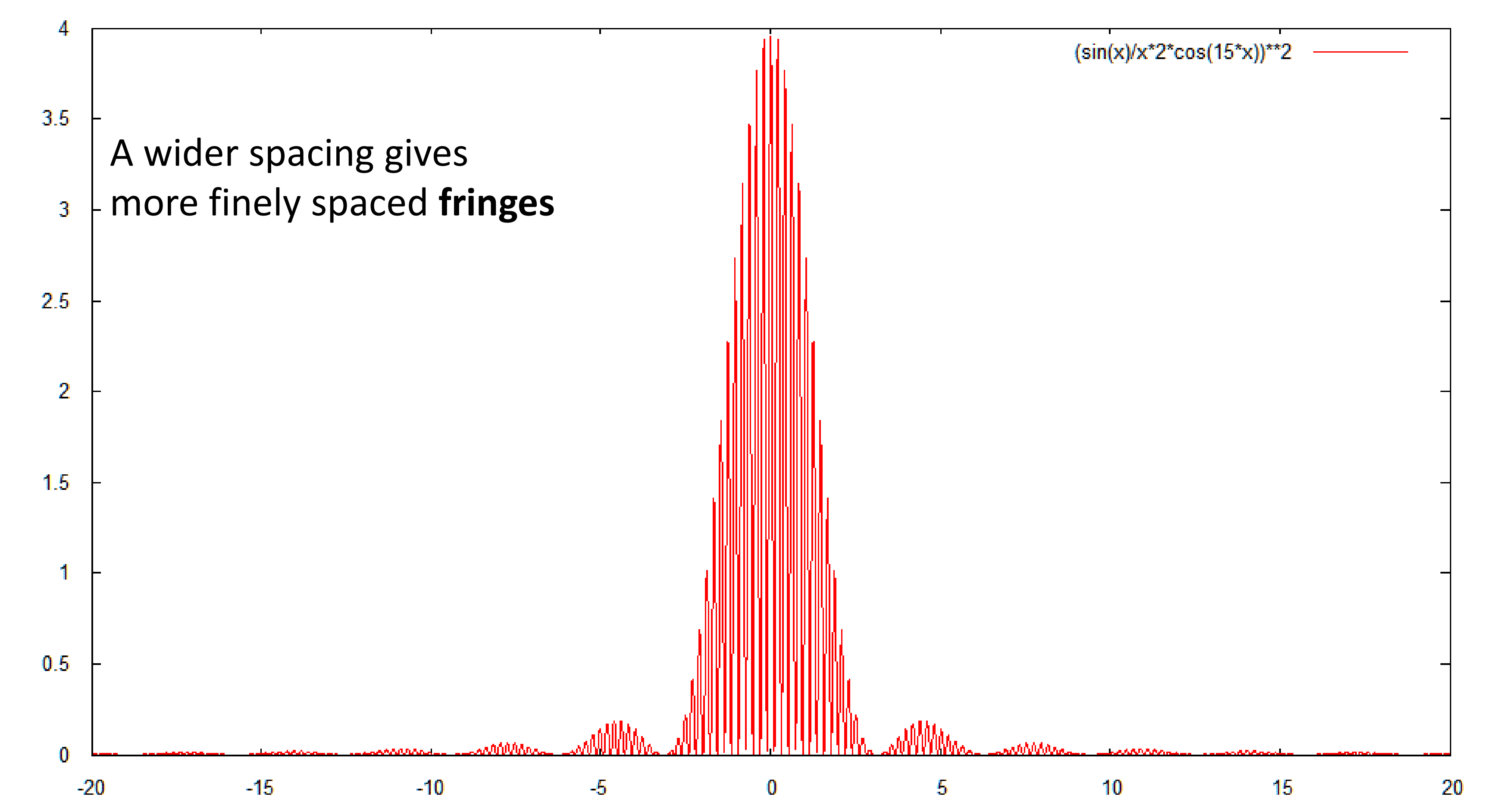
two-point interference   Single dish pattern

Intensity pattern for B = 5 * D



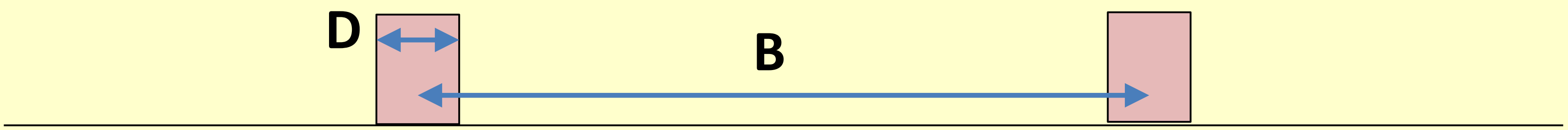
$k_x D \sin\alpha \propto \alpha$

B = 15 * D



$k_x D \sin\alpha \propto \alpha$

Case 3: two dishes with phase shift



g(x) as before, but phase shift φ between the two antennas

$$E(\alpha) = \frac{1}{D}\int_{-B/2-D/2}^{-B/2+D/2} e^{i\left(k_x x-\frac{\varphi}{2}\right)} dx + \frac{1}{D}\int_{B/2-D/2}^{B/2+D/2} e^{i\left(k_x x+\frac{\varphi}{2}\right)} dx$$

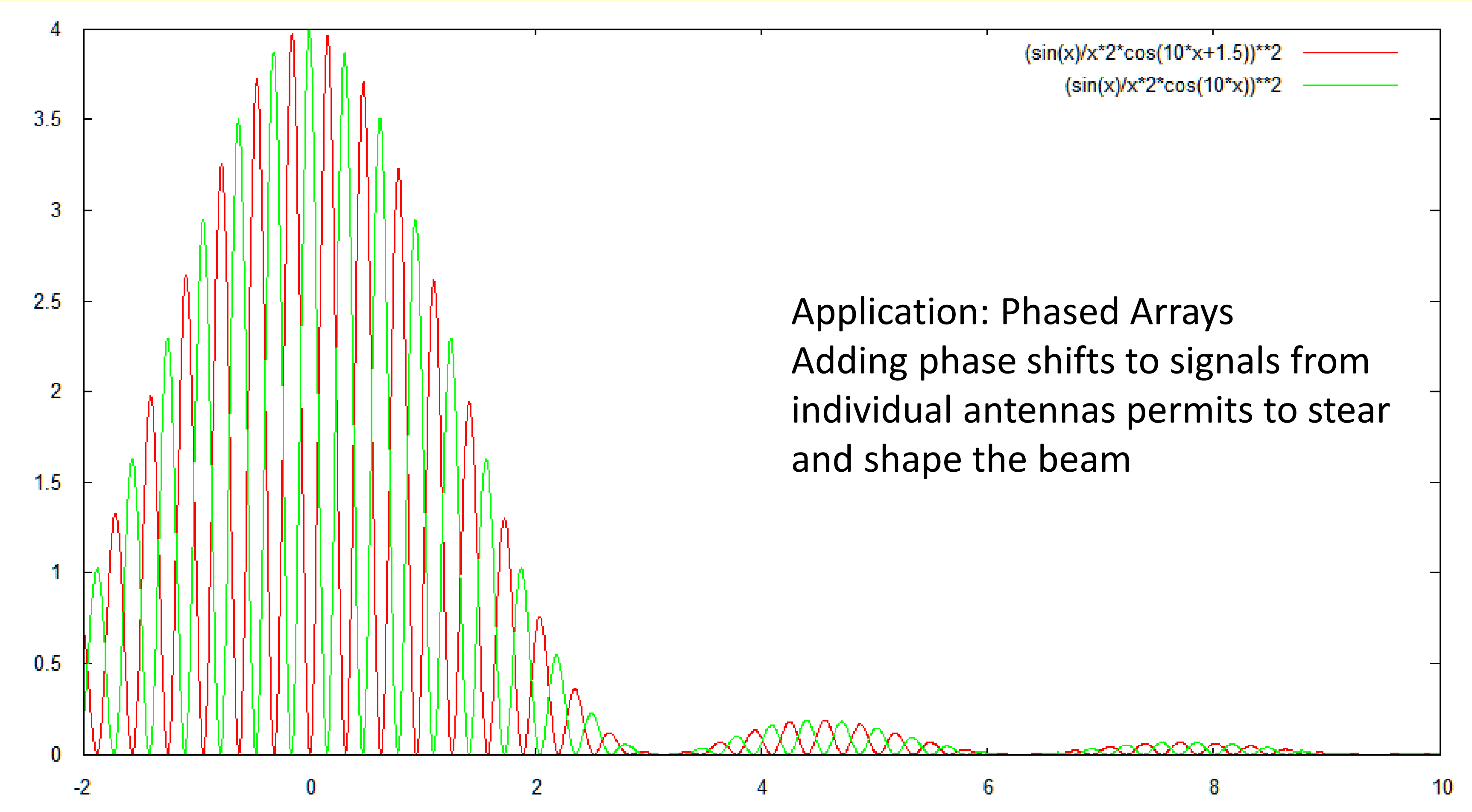
$$= \frac{e^{ik_x\left(-\frac{B}{2}+\frac{D}{2}\right)-\frac{i\varphi}{2}} - e^{ik_x\left(-\frac{B}{2}-\frac{D}{2}\right)-\frac{i\varphi}{2}}}{ik_x D} + \frac{e^{ik_x\left(\frac{B}{2}+\frac{D}{2}\right)+\frac{i\varphi}{2}} - e^{ik_x\left(\frac{B}{2}-\frac{D}{2}\right)+\frac{i\varphi}{2}}}{ik_x D}$$

$$= \left(e^{ik_x\frac{B}{2}+\frac{i\varphi}{2}} + e^{-ik_x\frac{B}{2}-\frac{i\varphi}{2}}\right) \frac{e^{ik_x\frac{D}{2}} - e^{-ik_x\frac{D}{2}}}{ik_x D}$$

$$Re\ E(\alpha) = \quad 2\cos\left(\frac{k_x B+\varphi}{2}\right) \quad * \quad \text{sinc}\left(k_x\frac{D}{2}\right)$$

Interference pattern | Single dish pattern

Phase shifts   shift the fringes



$k_x D \sin\alpha \propto \alpha$

$\mathcal{F}$ourier transform

- linear transformation between
  - time ⇔ frequency
  - space ⇔ spatial frequency (wave vector k)
  - f(t) ⇔ $\mathcal{f}(\omega)$
  - $\mathcal{F}(\alpha * f + g) = \alpha * \mathcal{F}(f) + \mathcal{F}(g)$
- convolution theorem:
  - $\mathcal{F}(f \otimes g) = \mathcal{F}(f) * \mathcal{F}(g)$

Properties of Fourier transform

- Small dish ⇐⇒ wide pattern (HPBW = 58°λ/D))

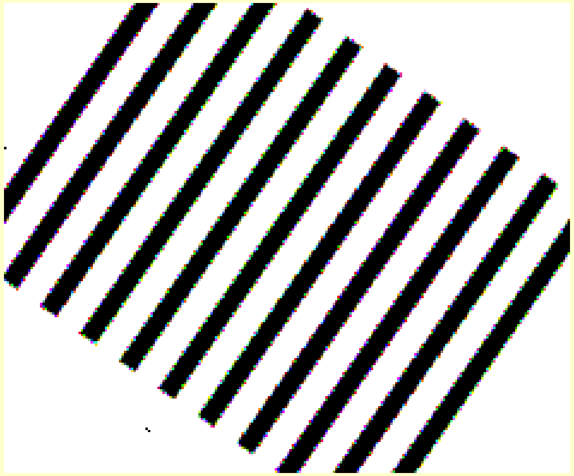
- Uniform illumination ⇐⇒ sinc(x) pattern
- Gaussian illumination ⇐⇒ Gaussian pattern (no sidelobes!!!)

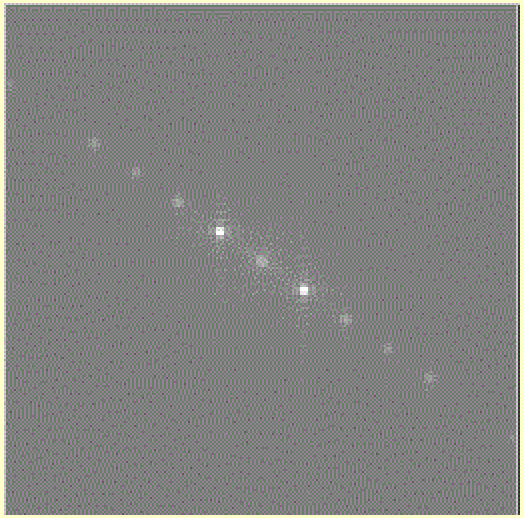
$$\sigma\_illumination * \sigma\_pattern = 1$$

Consequences for interferometers

- widely separated dishes ➔ finely spaced fringes
- few dishes (lower cost) ➔ many fringes (more difficult to interpret)

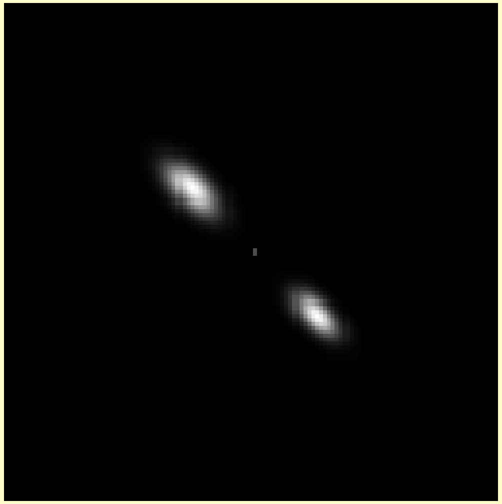
Fourier transform in 2D

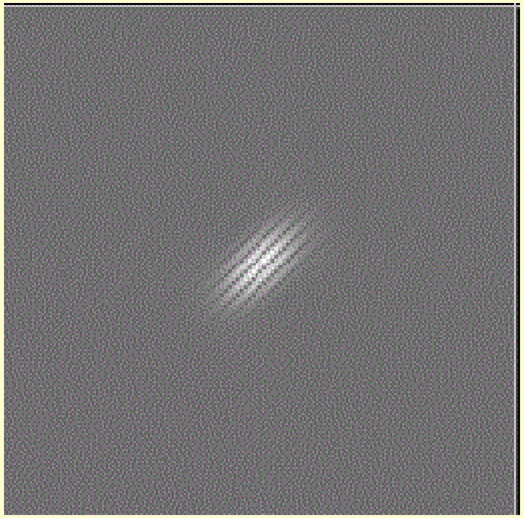




- Bars are long ➔ narrow spectrum along that direction
- Bars are thin ➔ broad spectrum
- Bars are evenly spaced, same shape ➔ spectral dots are well defined and evenly spaced (indicates the separation of the bars)
- Bars have sharp borders ➔ the spectral points have haloes

Fourier transform in 2D





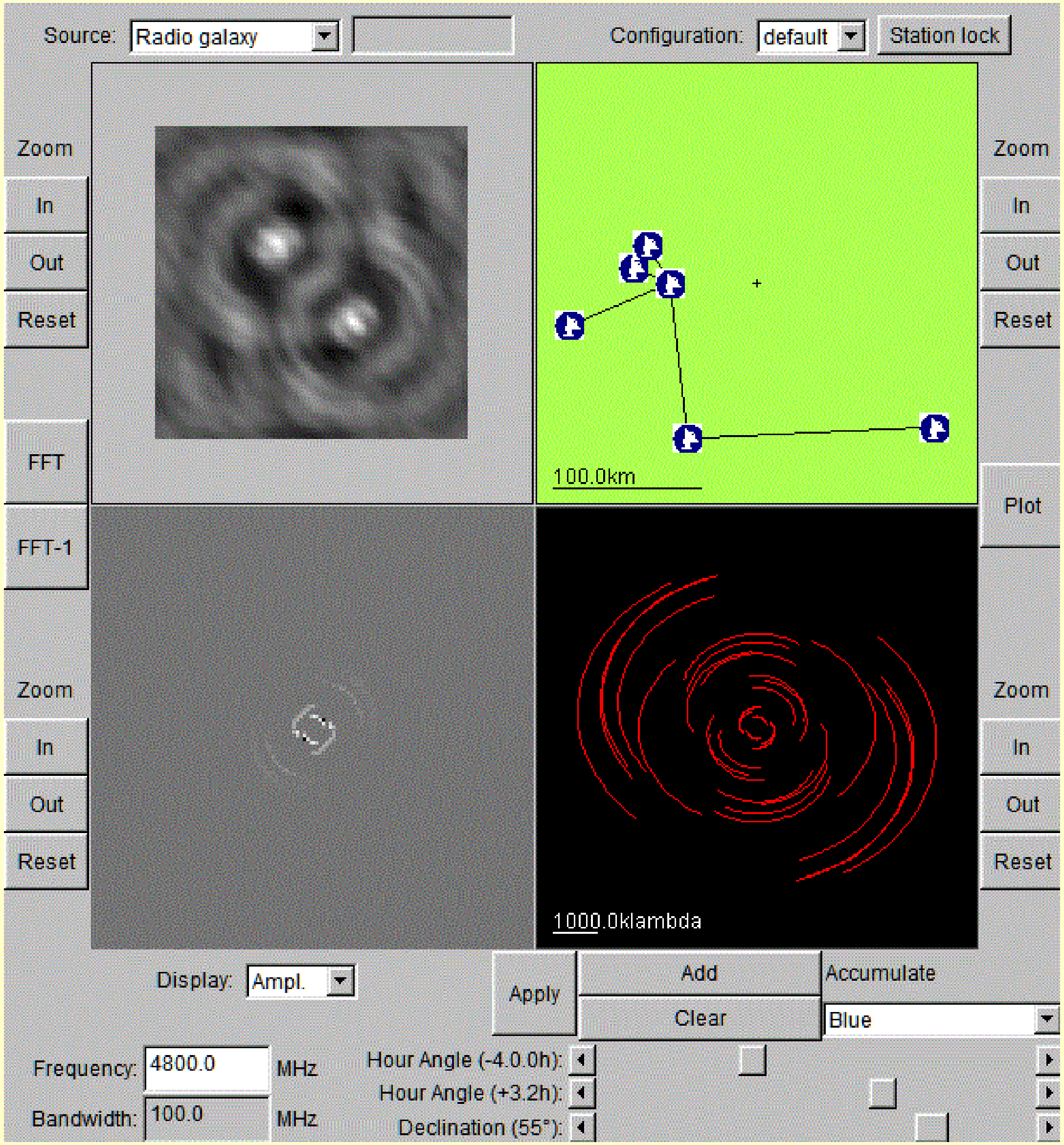
Radio galaxy

- Two blobs ➔ numerous fringes along their orientation (their spacing gives angular separation of blobs)
- Blobs are narrow ➔ spectrum is broader in the direction where the blobs are narrower

Aperture synthesis

- The longer the baseline, the finer are the structures an interferometer can detect: $\sin \Delta\alpha = \lambda\Delta\phi/B$
- A multiple antenna interferometer has several baselines of different length and direction. From the fringe pattern one can reconstruct the image (Fourier transform).
- As the Earth rotates during observation time, the projected baselines change, and thus provide more information
- Incomplete coverage of baselines causes artifacts in the reconstructed image

VirtualRadioInterferometer



http://astro.u-strasbg.fr/~koppen/vri.html

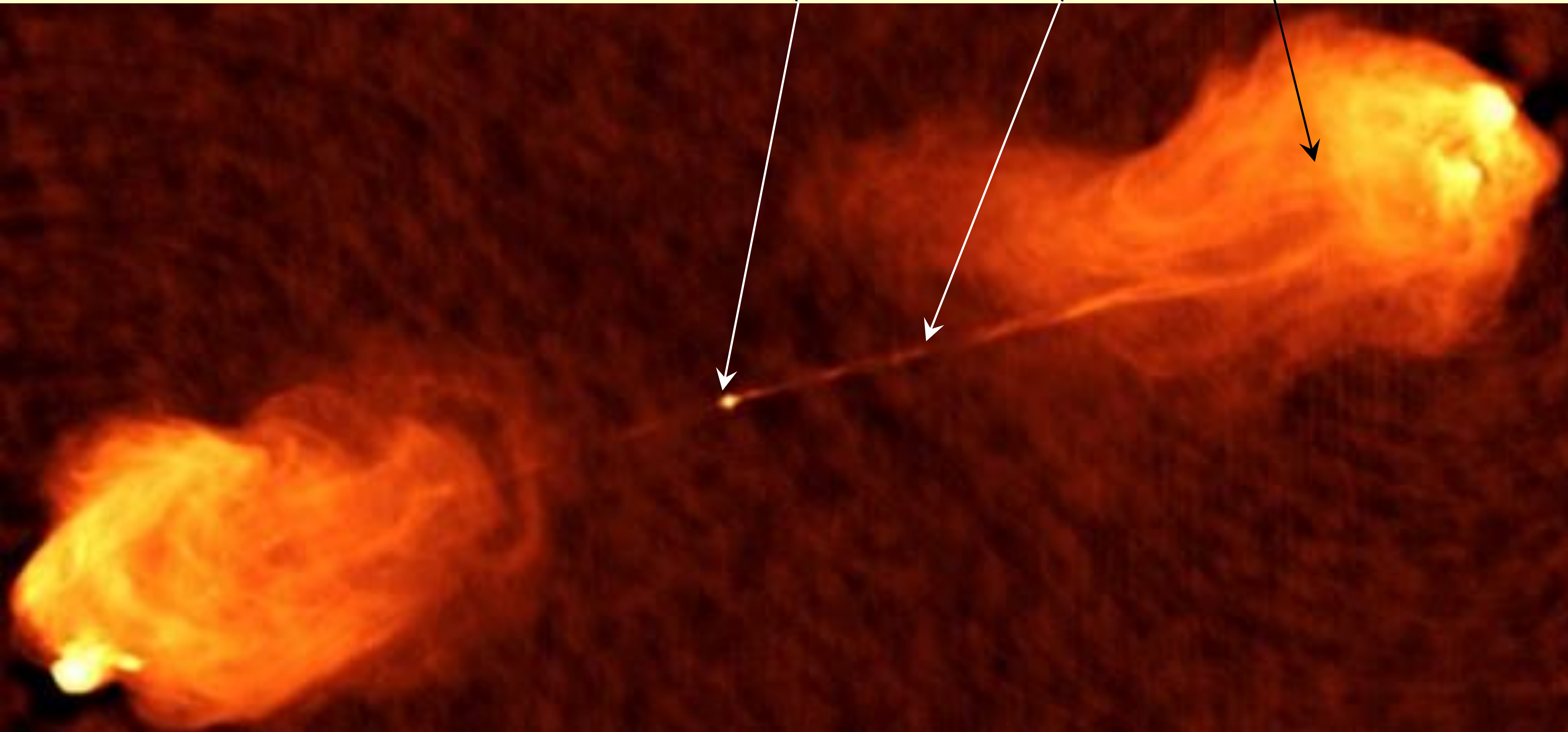
Very **L**arge **A**rray, Socorro, New Mexico


VLA

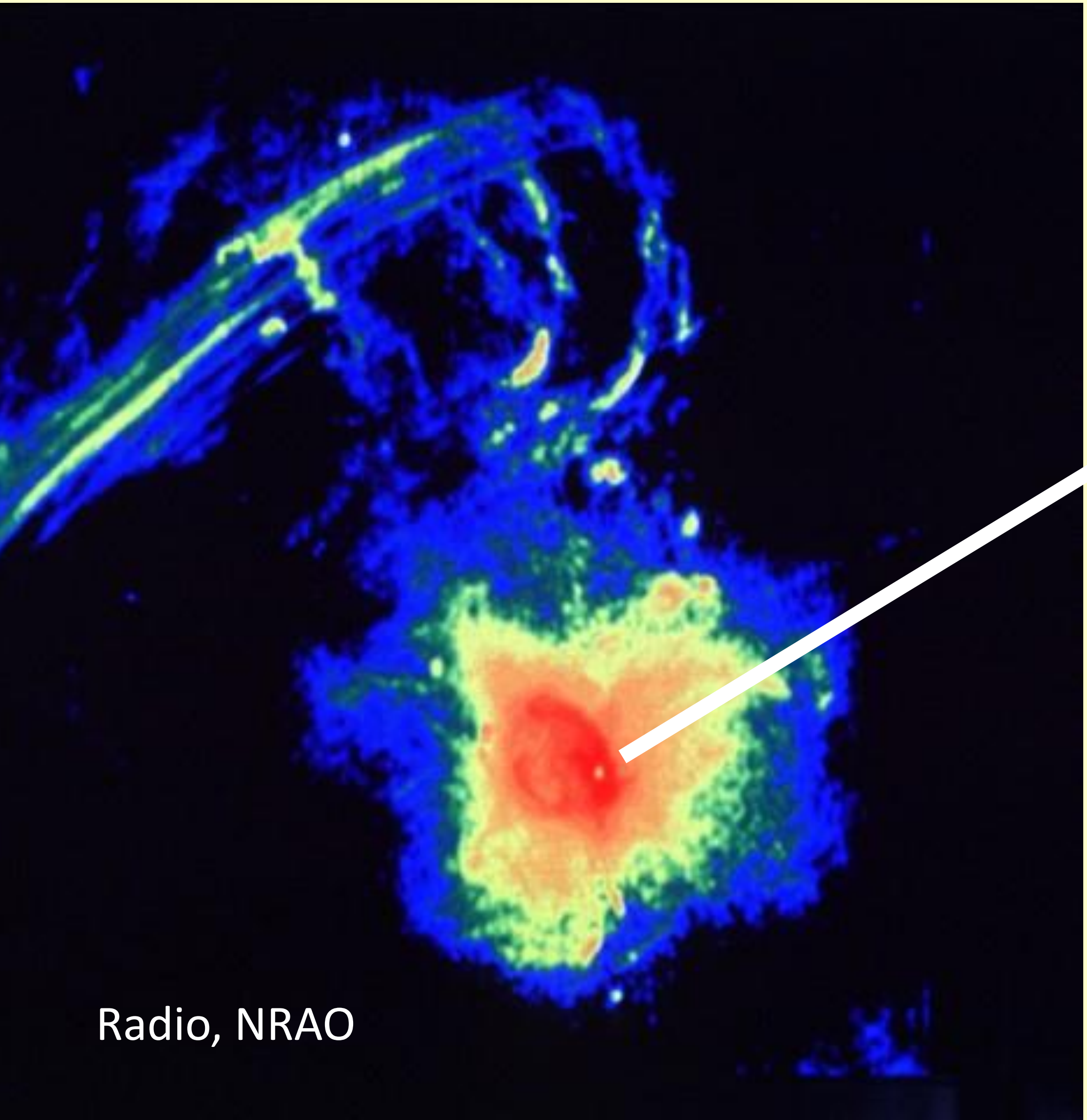
Cyg A is a radio galaxy

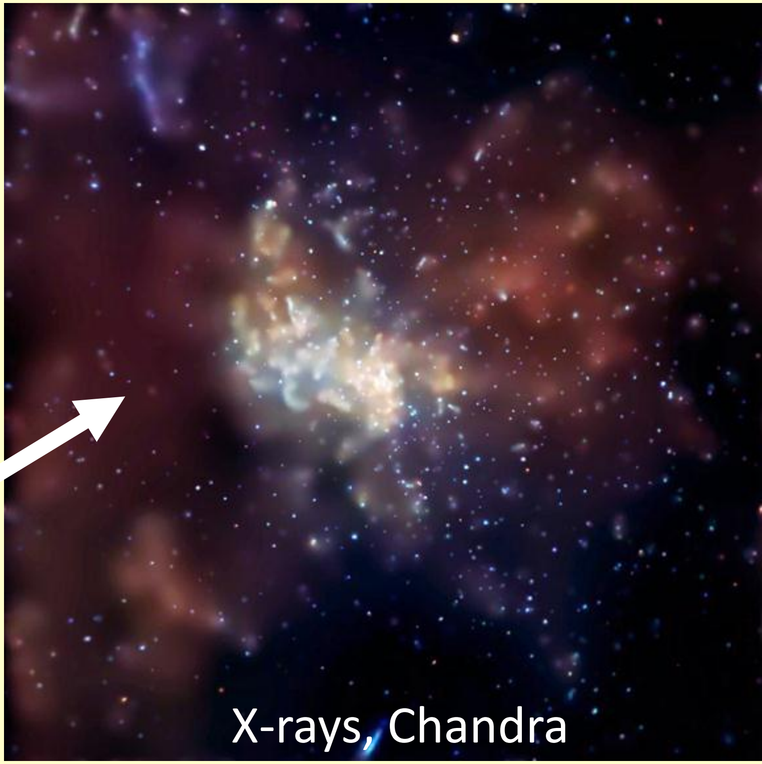
spewing out two jets of gas

which collide with intergalactic gas



Sgr A = the centre of our Milky Way





but Cas A = remnant of Supernova = exploded massive star

IR – Spitzer

Opt. – HST

Xray -- Chandra

Short list of Interferometers

- Westerbork (NL): 14x 25m E-W
- ATCA (Austral.): 6x 22m E-W

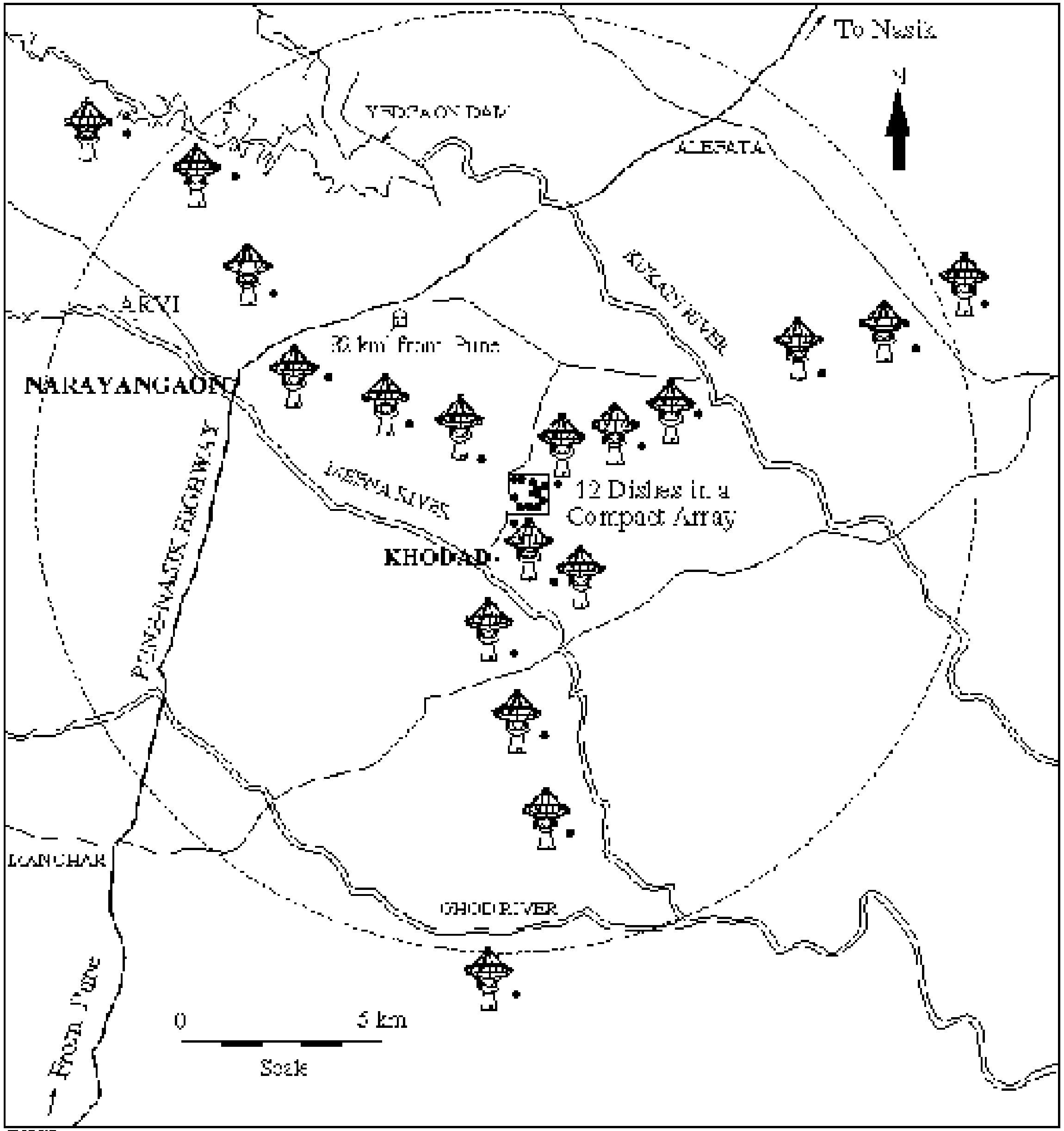
- VLA (NM, USA): 27x 25m Y
- GMRT (Pune, India): 30x 45m Y

- CARMA (CA, USA): 6x 10m (mmWave)
- IRAM (French alps): 6x 15m (mmWave)
- SMA (Mauna Kea): 8x 6m (<1000 GHz)

Giant **M**etrewave **R**adio **T**elescope, Pune

30x 45m diam
baseline < 25 km

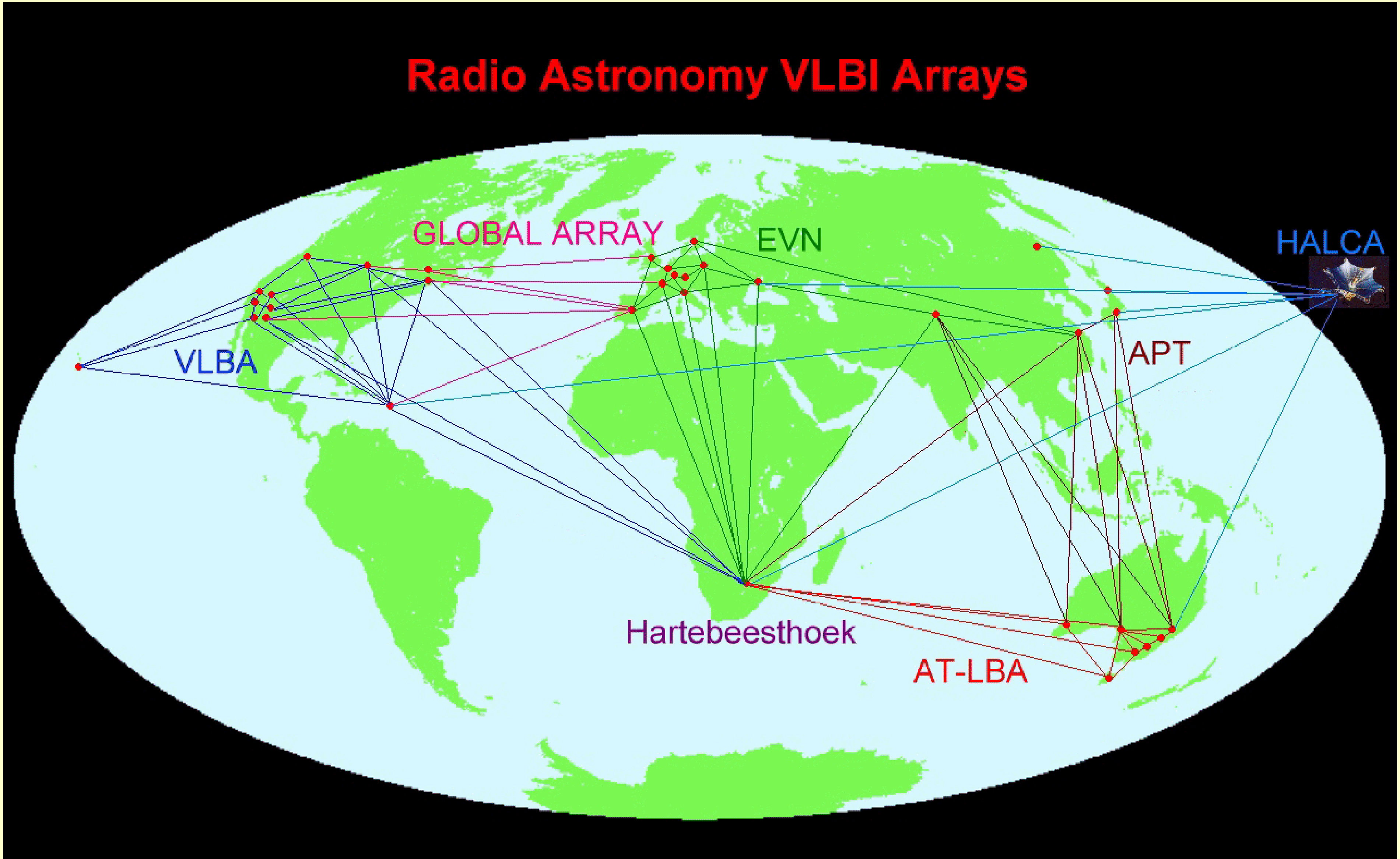
LOCATIONS OF GMRT ANTENNAS (30 dishes)



Problem No.3: Phase stability

- The receivers of an interferometer must preserve the phase of the signal ➔ all local oscillators must be phase-locked to each other, and preferably to a stable master oscillator (atomic clock).

Very Long Baseline Interferometry

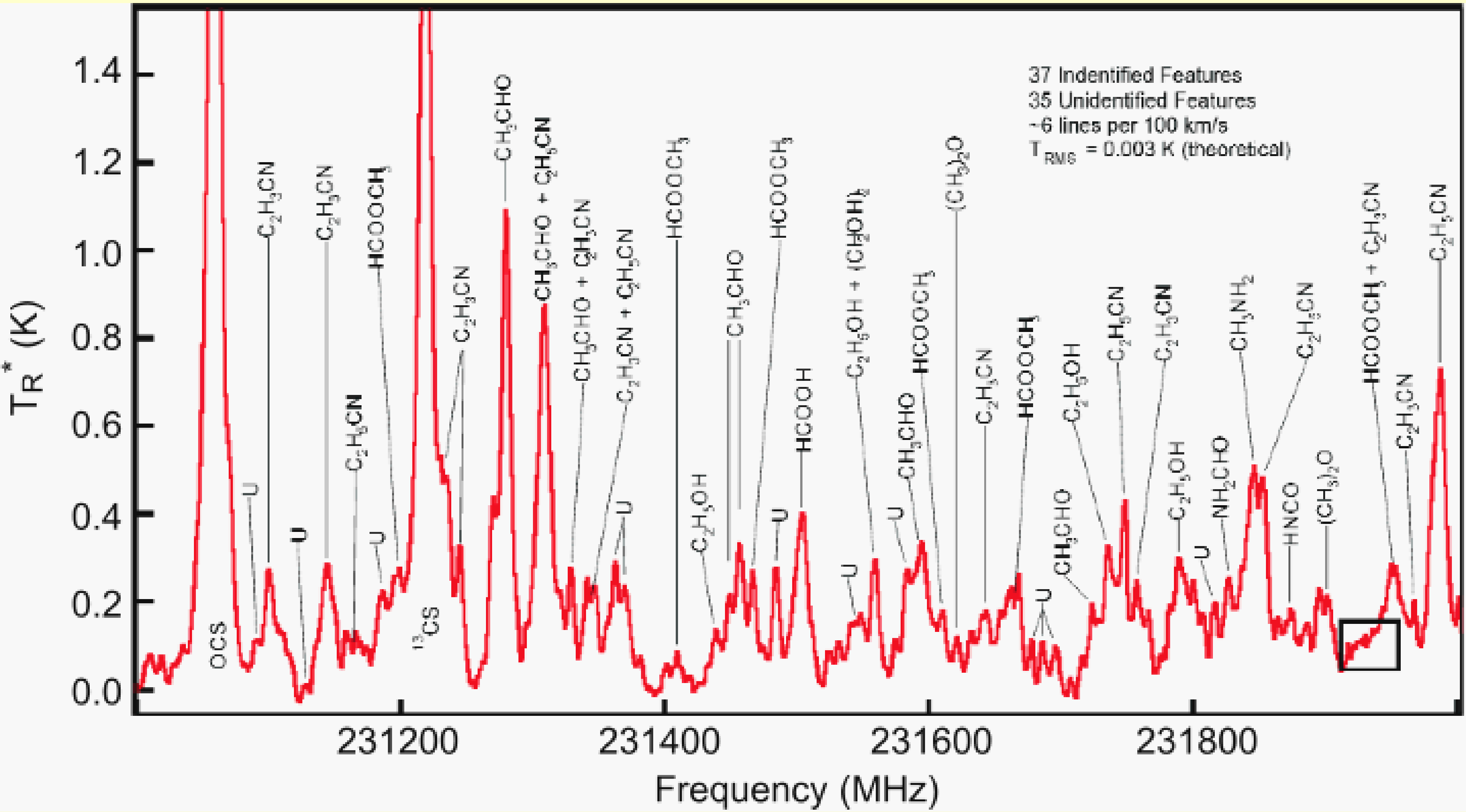


What lies ahead? (I)

- (sub-)Millimetre waves (above 30 GHz)
  - Molecular lines
    - cool, star-forming gas clouds
    - solar systems in formation
    - Extra-solar planets (atmospheres)
- Needs very dry skies:
- **A**tacama**L**arge**M**illimetre**A**rray
  - 30 … 1000 GHz, 64 antennas 12m; 5059m altitude

    first light: Oct.2011

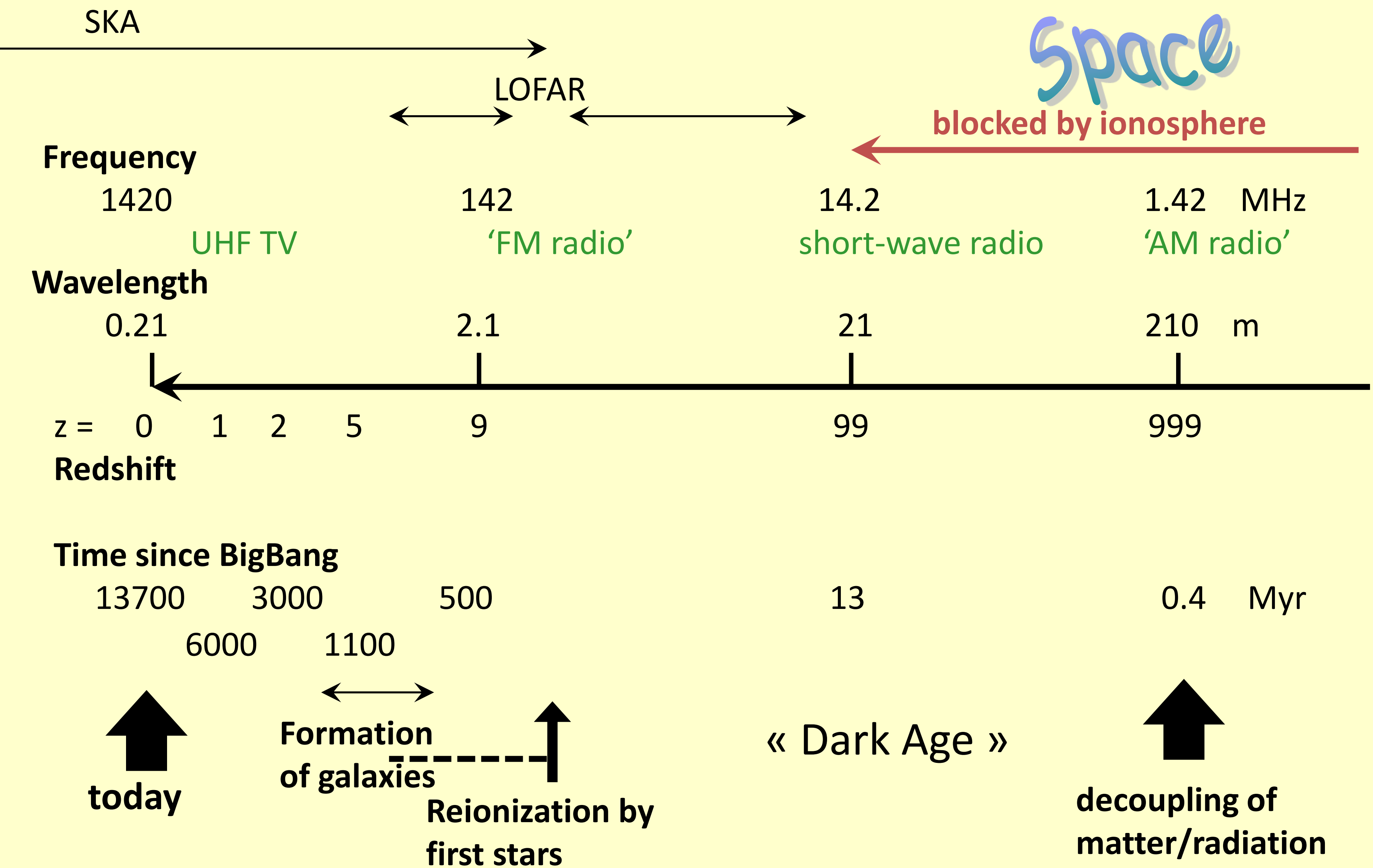

ALMA/NAOJ

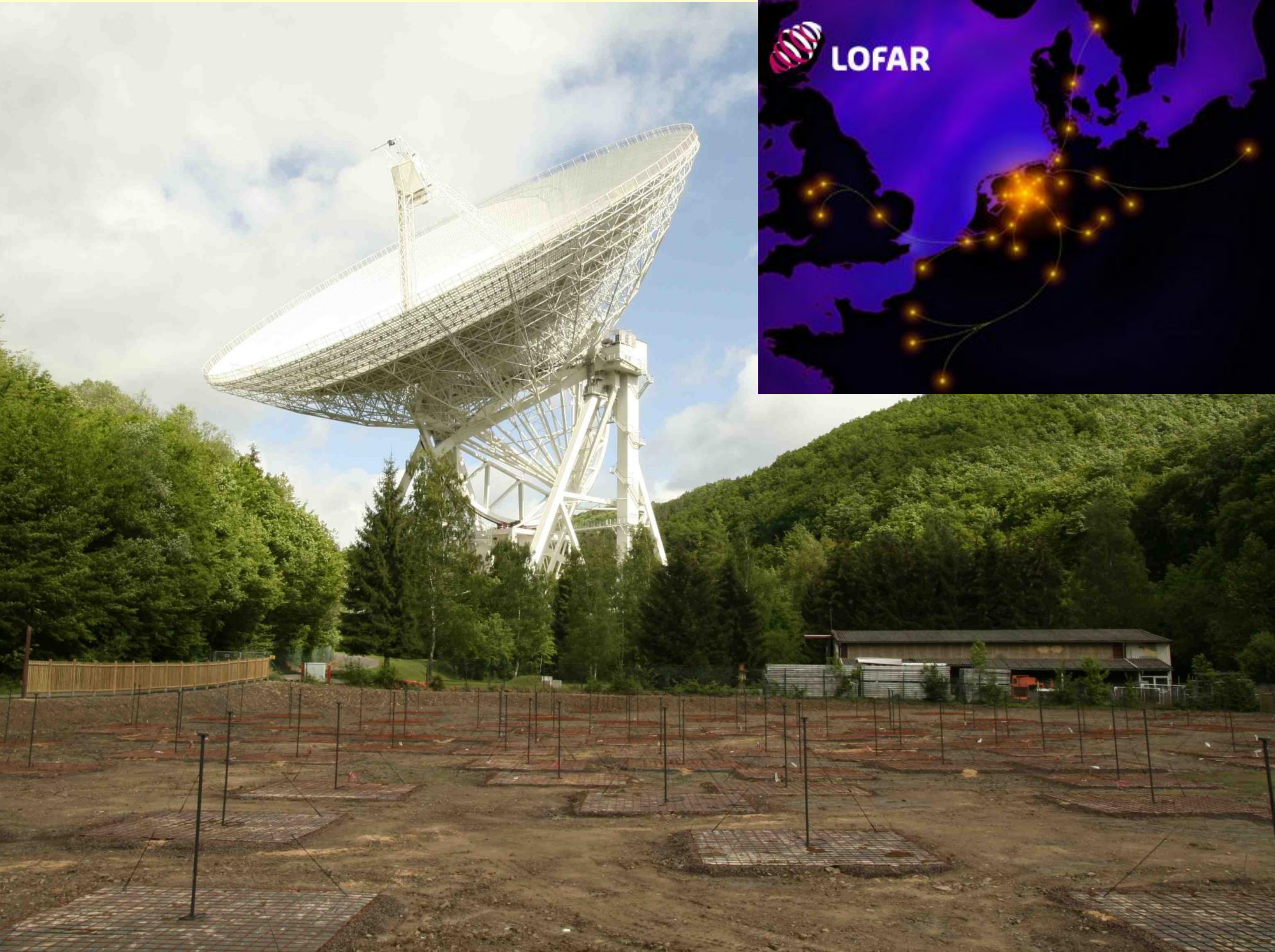

37 Indentified Features
35 Unidentified Features
~6 lines per 100 km/s
T_{RMS} = 0.003 K (theoretical)
T_R^* (K)
Frequency (MHz)
231200
231400
231600
231800
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
OCS
^{13}CS
U
C_2H_5CN
$HCOOCH_3$
CH_3CHO
$CH_3CHO + C_2H_5CN$
$CH_3CHO + C_2H_3CN$
$C_2H_3CN + C_2H_5CN$
C_2H_3CN
C_2H_5OH
HCOOH
$C_2H_5OH + (CH_2OH)_2$
$(CH_3)_2O$
NH_2CHO
CH_3NH_2
HNCO
$HCOOCH_3 + C_2H_3CN$

What lies ahead? (II)

- Low frequencies (below 100 MHz)
  - Red-shifted HI 21 cm line from very early universe: forming galaxies
  - … ???
- **LO**w**F**requency**AR**ray (Netherlands → NEurope)
  - 30…80 MHz, 120…240 MHz, phased array 93 stations with 100 antennas (simple dipoles) each, operational
- **S**quare**K**ilometre**A**rray (Australia,SAfrica)
  - 0.1 … 25 GHz, several 1 km^2 area stations 3000 km apart, <0.1’’ at 1.4GHz, site sel.2012, oper.2020?

HI 21 cm line from early Universe

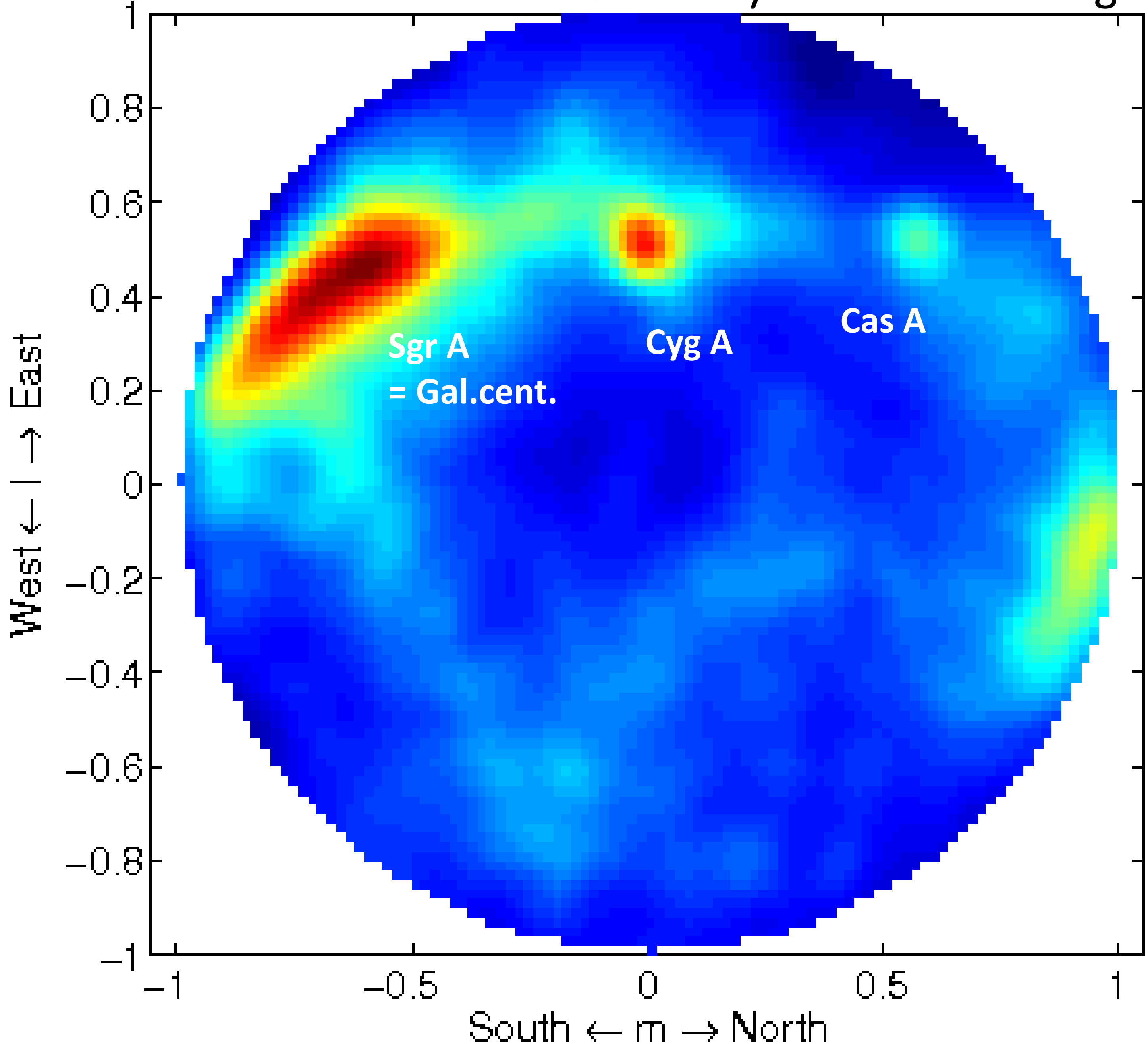



LOFAR

LOFAR et al.

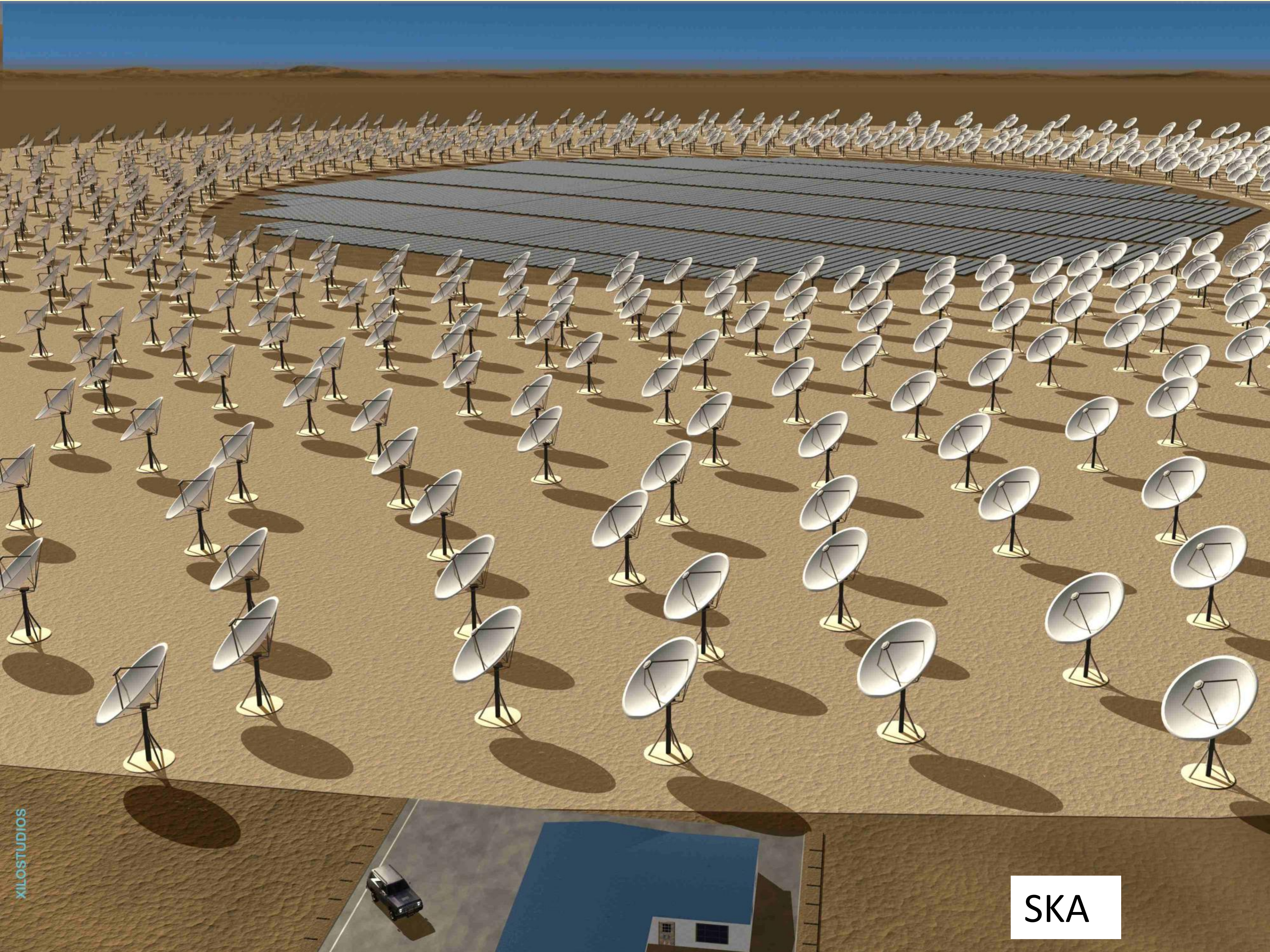
- The signals from **all** antennas (simple dipoles) at **all** stations are digitized and stored, including information on polarization
- Software processing:
  - selection of frequency
  - combination with phase shifts to create antenna beams
  - to suit any objectives

LOFAR sky above Effelsberg



29 oct. 2007

42 MHz


XILOSTUDIOS
SKA