# TWO FLUID GAS AND DUST MIXTURES IN SPH


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One of the most challenging aspects of numerical simulations of dust and gas mixtures is the often enormous difference between gas and dust evolutionary time scales. Whenever the dust stopping time becomes much smaller than the gas evolutionary time scale, explicit/implicit integration schemes require an excessively large number of time-steps, and none of the previous SPH schemes in the two fluid approach (Monaghan & Kocharyan 1995 and 1997; Maddison, Humble & Murray 2003; Rice et al. 2004; Barriere-Fouchet et al. 2005, Laibe & Price 2012) have addressed this problem. In the present work, a method to avoid the time integration of the small dusty grains evolution equations, in the Smoothed Particle Hydrodynamics (SPH) two fluid approach is proposed. By assuming a very simple exponential decay model for the relative velocity between the gas and dust components, all the effective characteristics of the drag force can be reproduced. Taking as a reference the recent work of Laibe & Price (2011/12) a series of tests have been performed in order to compare the accuracy of the present method with a standard integration method (Loren-Aguilar & Bate 2014).


- The effect of the drag force can be modeled as an exponential decay of the relative velocity between the dust and gas components (Loren-Aguilar & Bate 2014)

$$\vec{v}_D^{\,i}(t+\delta t) = \vec{v}_D^{\,i}(t) - \frac{\nu}{N_i}\sum_k^{Gas} m_k \frac{\xi_{ik}}{\rho_k}(\vec{v}_{ik}\cdot\hat{r}_{ik})\hat{r}_{ik}D(h_k),$$

$$\vec{v}_G^{\,j}(t+\delta t) = \vec{v}_G^{\,j}(t) + \frac{\nu}{N_i}\sum_k^{Dust} m_k \frac{\xi_{kj}}{\rho_j}(\vec{v}_{kj}\cdot\hat{r}_{kj})\hat{r}_{kj}D(h_j),$$

$$\xi_{ij} = \frac{\rho_i}{\rho_i+\rho_j}(1-e^{-\delta t/t_s}),$$

where $t_s$ is the stopping time

$$t_s \equiv \frac{\hat{m}_D}{K_s(1+\rho_D/\rho_G)},$$

$\hat{m}_D$ is the dust grain mass, $K_s$ is the drag force coefficient, $N_i$ is a normalization factor used to avoid fluctuations in the drag force if a low number of neighbours is present (Randles & Libersky 1996) and is given by

$$N_i = \sum_k^{Gas} \frac{m_k}{\rho_k} D(h_k).$$

D(h) is the double hump cubic kernel (Fulk & Quinn 1996), ν corresponds to the number of spatial dimensions of the system, $m_k$ is the mass of the SPH particle, $h_k$ is the SPH particle smoothing length, and ρ corresponds to the SPH particle density.

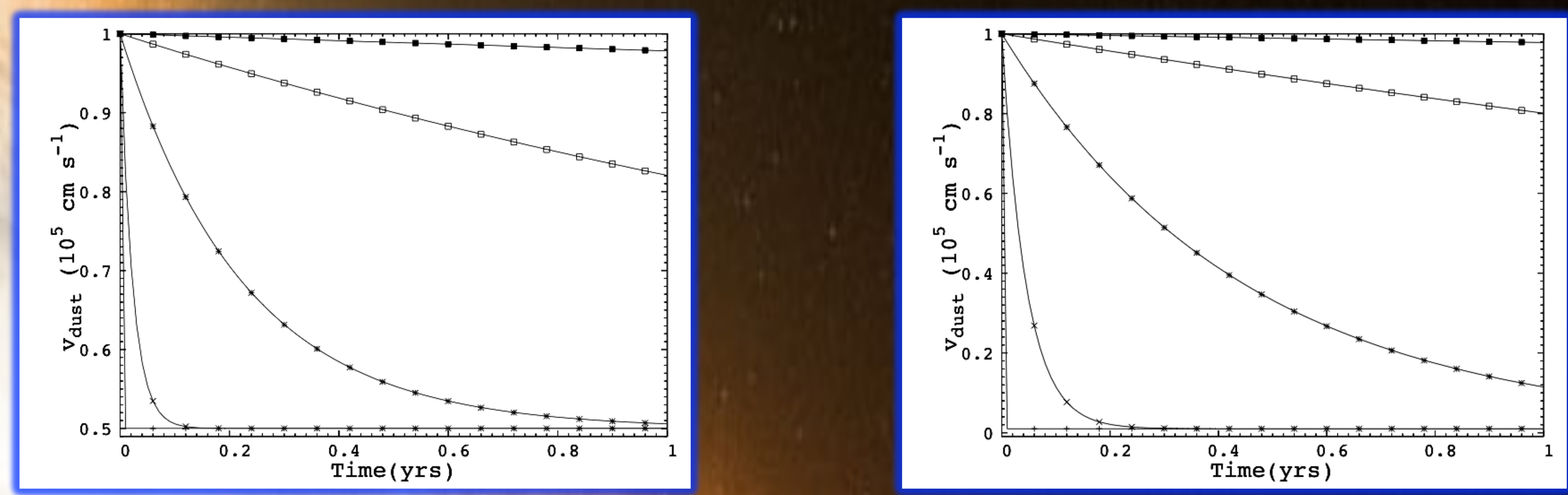
- The previous equations constitute an approximate solution for the Euler's equations of a gas/dust mixture, regardless of drag's strength. Errors no bigger than a 1% have been found in the approximation for the performed tests.

- Exact linear and angular momentum conservation is guaranteed by the scheme.

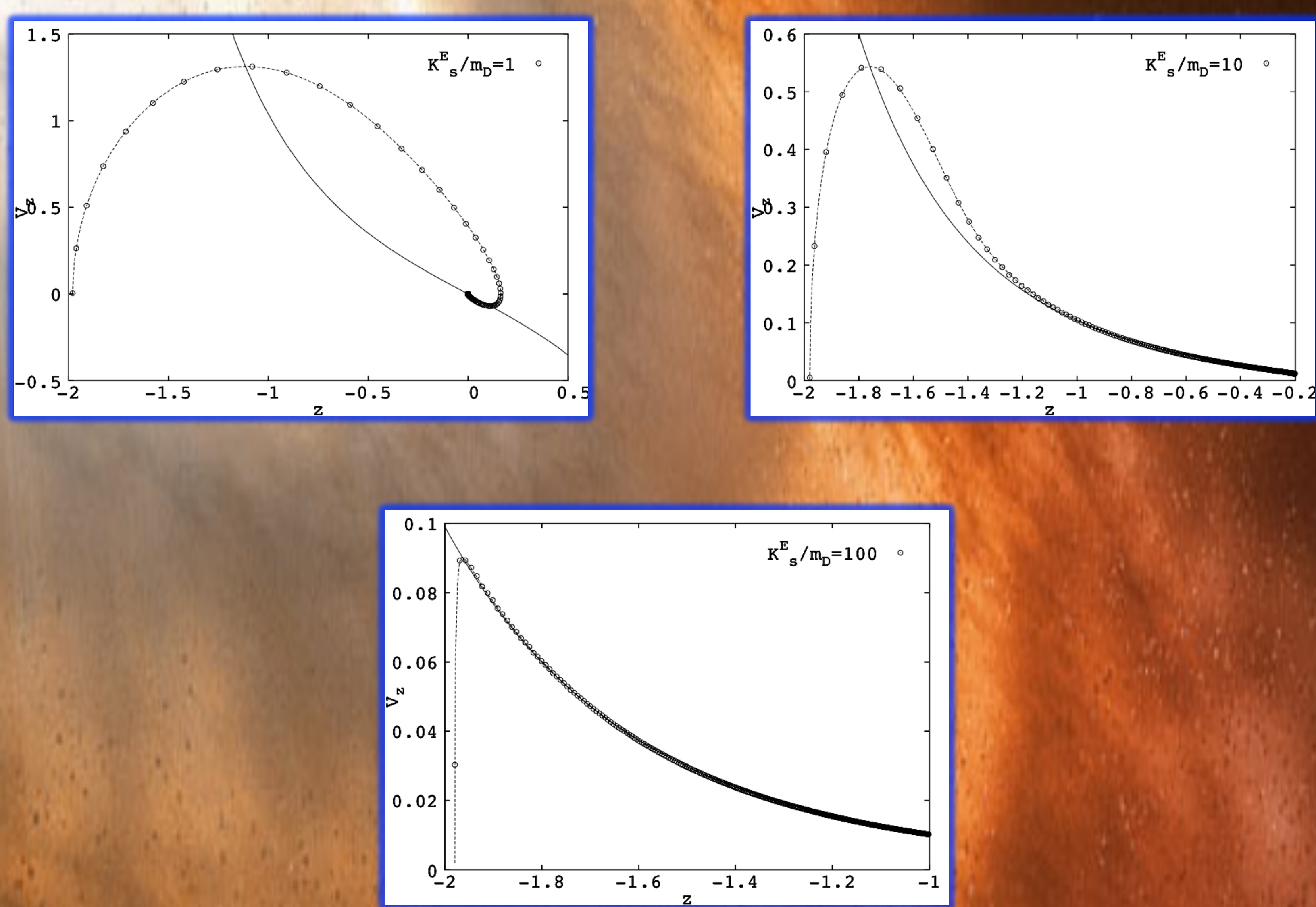
- The method has been able to obtain at least, in all of the tested cases, the same results as the explicit/implicit integration scheme of Laibe & Price (2012).

- The system is evolved using exclusively the gas Courant time condition. Thus, it avoids the need for acceleration recalculations, being many times faster than implicit/explicit methods for highly coupled dust grains.

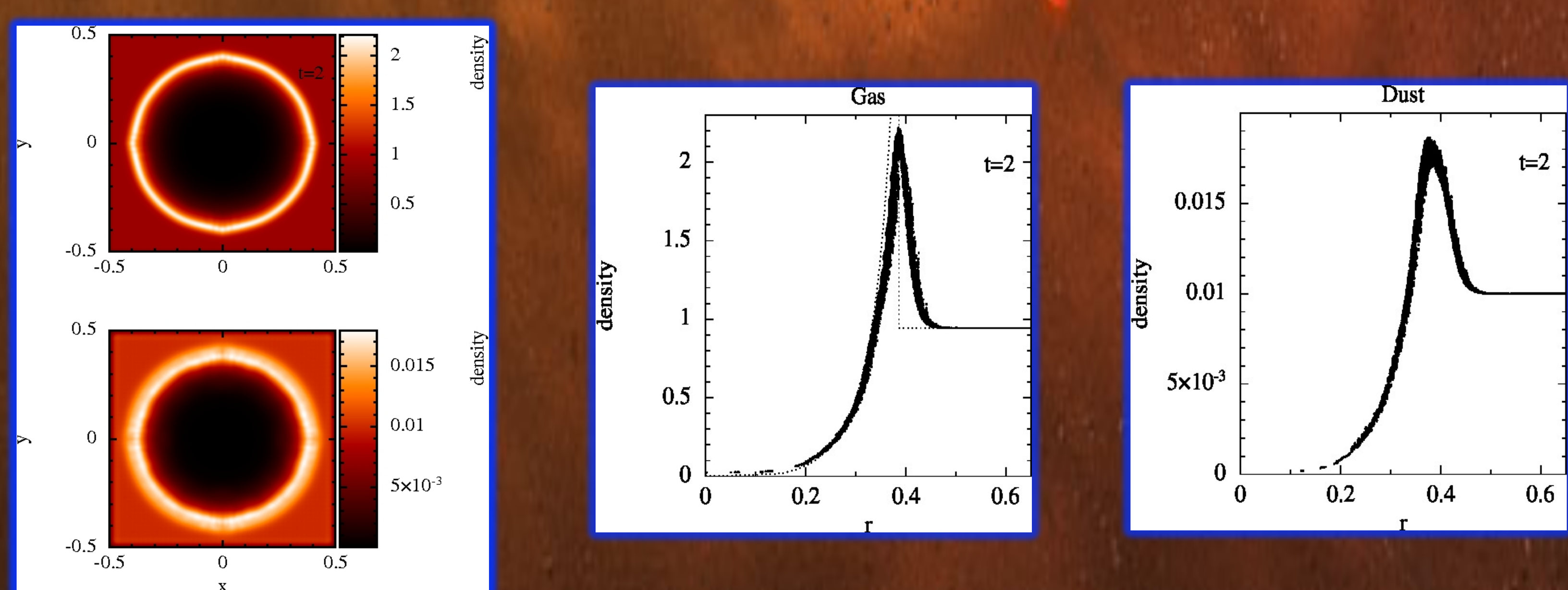
- As any SPH two fluid method, it produces an excess of dissipation when drag forces are very intense and dust-to-gas ratios are close to unity. However, the method produces much less dissipation than the previous methods.



- Time evolution of the velocity of the dust component in a three dimensional gas/dust mixture in the Epstein regime, for several different grain sizes: *s*= 1 mm, 1 m, 10 m, 100 m, and 1 km from bottom to top. The adopted physical conditions are those appropriate for a dust particle at the mid-plane of a protoplanetary disc at 1 au: $\rho_G = 10^{-9}$ g cm$^{-3}$, $v_{th} \approx 10^5$ cm s$^{-1}$, and $\rho_D = 3$ g cm$^{-3}$. The computational domain comprises a total volume of 1 cubic au. The method has been tested with two different dust-to-gas ratios, $\rho_D/\rho_G = 1$ (left-hand figure), and $\rho_D/\rho_G = 0.01$ (right-hand figure). A total of $20^3$ gas and $20^3$ dust particles have been used for the test. Solid lines represent the analytical solutions for the problem for each dust grain size (Loren-Aguilar & Bate 2014).



- Settling velocity of a dust particle as a function of z, for different drag intensities, in a one dimensional vertical section of an isothermal disk. Dashed lines correspond to the numerical solution of the problem, while solid lines represent the limiting velocity for each case. As can be seen, the higher the drag intensity is, the sooner the limiting velocity is reached, as expected (Loren-Aguilar & Bate 2014).



- Result of a three-dimensional Sedov problem with a dust-to-gas ratio of 0.01 in a high drag regime. Dashed lines represent the self-similar solution of the Sedov problem. In this case, having used a total number 50x50x50 particles, no over dissipation is appreciated due to the low dust-to-gas ratio (Loren-Aguilar & Bate 2014).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