Secular evolution of “wave” patterns in circumbinary planetesimal disks

Tatiana V. Demidova*, **Ivan I. Shevchenko**

*e-mail: proxima1@list.ru

1 Introduction

Planet formation scenarios and the observed planetary dynamics in binaries pose a number of theoretical challenges, especially in what concerns circumbinary planetary systems. We explore the dynamical stirring of a planetesimal circumbinary disk in the epoch when the gas component disappears. The orbital precession of the particles forming the disk generates prominent transient structures. Using a secular theory, we describe these structures analytically.

2 Secular evolution

We consider a binary star with a circumbinary disk of planetesimals; m_1 and m_2 are the masses of the binary components, a_b and e_b are the semimajor axis and eccentricity of the binary's orbit, a is the semimajor axis of a planetesimal orbit. All masses are expressed in $M_\odot$, distances in AU, time in yr.

We extend and refine the theory by Moriwaki and Nakagawa [1] for the secular dynamics of planetesimals in circumbinary disks, combining the approach of Ref. [1] with that of Ref. [2], where the circumstellar case was considered. The resulting formulas for the secular evolution of the eccentricity e and the longitude of pericenter ϖ of a circumbinary planetesimal are:

$$e = e_\mathrm{max}\left|\sin\frac{ut}{2}\right|, \quad \tan\varpi = -\frac{\sin ut}{1-\cos ut}, \tag{1}$$

where t is time, $e_\mathrm{max} = 2e_\mathrm{f}$, and e_f is the forced eccentricity,

$$e_\mathrm{f} = \frac{5}{4}\frac{(m_1-m_2)}{(m_1+m_2)}\frac{a_\mathrm{b}}{a}e_\mathrm{b}\frac{\left(1+\frac{3}{4}e_\mathrm{b}^2\right)}{\left(1+\frac{3}{2}e_\mathrm{b}^2\right)}; \tag{2}$$

$$u = \frac{3\pi}{2}\frac{m_1m_2}{(m_1+m_2)^{3/2}}\frac{a_\mathrm{b}^2}{a^{7/2}}\left(1+\frac{3}{2}e_\mathrm{b}^2\right). \tag{3}$$

Using a new variable $y = \frac{ut}{2}$, Eq. (2) can be rewritten:

$$\begin{aligned}
&\text{if } y \geq -\pi \text{ and } y \leq -\frac{\pi}{2}, \text{ then } \varpi = y + 5\frac{\pi}{2};\\
&\text{if } y \geq -\frac{\pi}{2} \text{ and } y \leq 0, \text{ then } \varpi = y + \frac{\pi}{2};\\
&\text{if } y \geq 0 \text{ and } y \leq \frac{\pi}{2}, \text{ then } \varpi = y + 3\frac{\pi}{2};\\
&\text{if } y \geq \frac{\pi}{2} \text{ and } y \leq \pi, \text{ then } \varpi = y - \frac{\pi}{2}.
\end{aligned} \tag{4}$$

Thus u can be regarded as a “precession rate” of an individual planetesimal orbit.

3 Theory versus numerics

As an actual binary star example, we consider *Kepler*-16. Its parameters are: $m_1 = 0.69M_\odot$, $m_2 = 0.2026M_\odot$, $e_\mathrm{b} = 0.159$, $a_\mathrm{b} = 0.2243$ AU [3]. The eccentricity and longitude of pericenter of the planetesimals (initially in circular orbits), as semimajor axis functions given by Eqs. (1) and (4) at $t = 10^5$ yr, are presented in Figure 1. Radial “waves” of the eccentricity, well-known from numeric diagrams in Refs. [1, 4, 5], are evident. The analytic “waves” closely match their numeric counterparts.

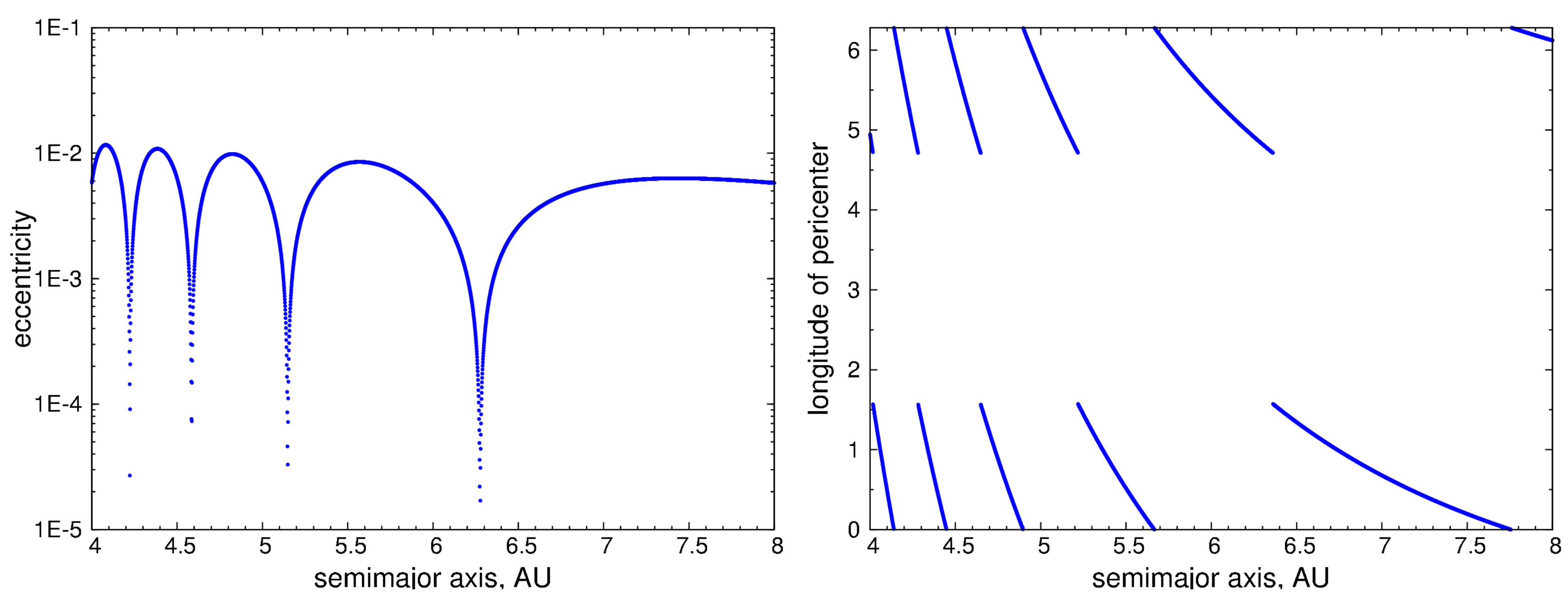

Figure 1: The planetesimal eccentricity (left) and longitude of pericenter (right) in function of the semimajor axis.

In Figure 2, analytical curves are depicted for the eccentricity and longitude of pericenter of the circumbinary planet *Kepler*-16b ($a = 0.7016$) in function of time. The secular curves perfectly match the corresponding numerical-experimental plots constructed in Refs. [6, 7].

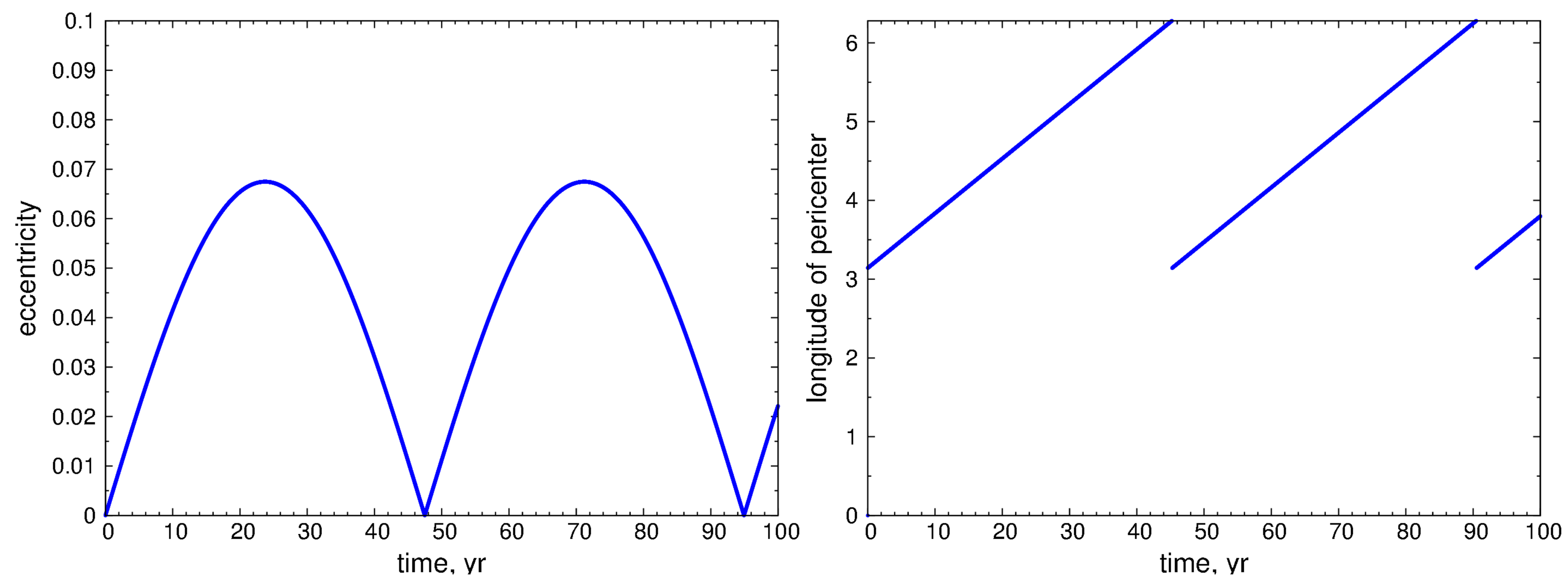

Figure 2: The secular eccentricity oscillation (left) and the longitude-of-pericenter rotation (right) of the planet *Kepler*-16b.

4 Spiral pattern

Formation of spiral density waves in astrophysical disks due to differential precession of orbits was considered earlier in Refs. [8, 9] in different settings. We investigate the circumbinary disk structure in the gas-free case. Using our secular theory, we deduce the analytical formula for the spiral pattern:

$$r(\theta) = \left(\frac{At}{\theta}\right)^{2/7} + B(1-\cos\theta), \tag{5}$$

$$A = \frac{3\pi}{2}\frac{m_1m_2}{(m_1+m_2)^{3/2}}a_\mathrm{b}^2\left(1+\frac{3}{2}e_\mathrm{b}^2\right), \quad B = \frac{5}{4}\frac{m_1-m_2}{m_1+m_2}a_\mathrm{b}e_\mathrm{b}\frac{\left(1+\frac{3}{4}e_\mathrm{b}^2\right)}{\left(1+\frac{3}{2}e_\mathrm{b}^2\right)}. \tag{6}$$

Thus the density wave is described by a shifted *lituus*.

Fixing the time of evolution to $t = 10^4$ binary periods, we analytically calculate the evolved planetesimal orbits in the semimajor axis a interval from $3a_\mathrm{b}$ to $16a_\mathrm{b}$. The model parameters are: $m_1 = M_\odot$, $m_2 = 0.2M_\odot$, $e_\mathrm{b} = 0.4$, $a_\mathrm{b} = 1$ AU. In Figure 3, the planetesimal orbits are shown as dotted curves. Results of an SPH-code numerical simulation, corresponding to this model, are also presented.

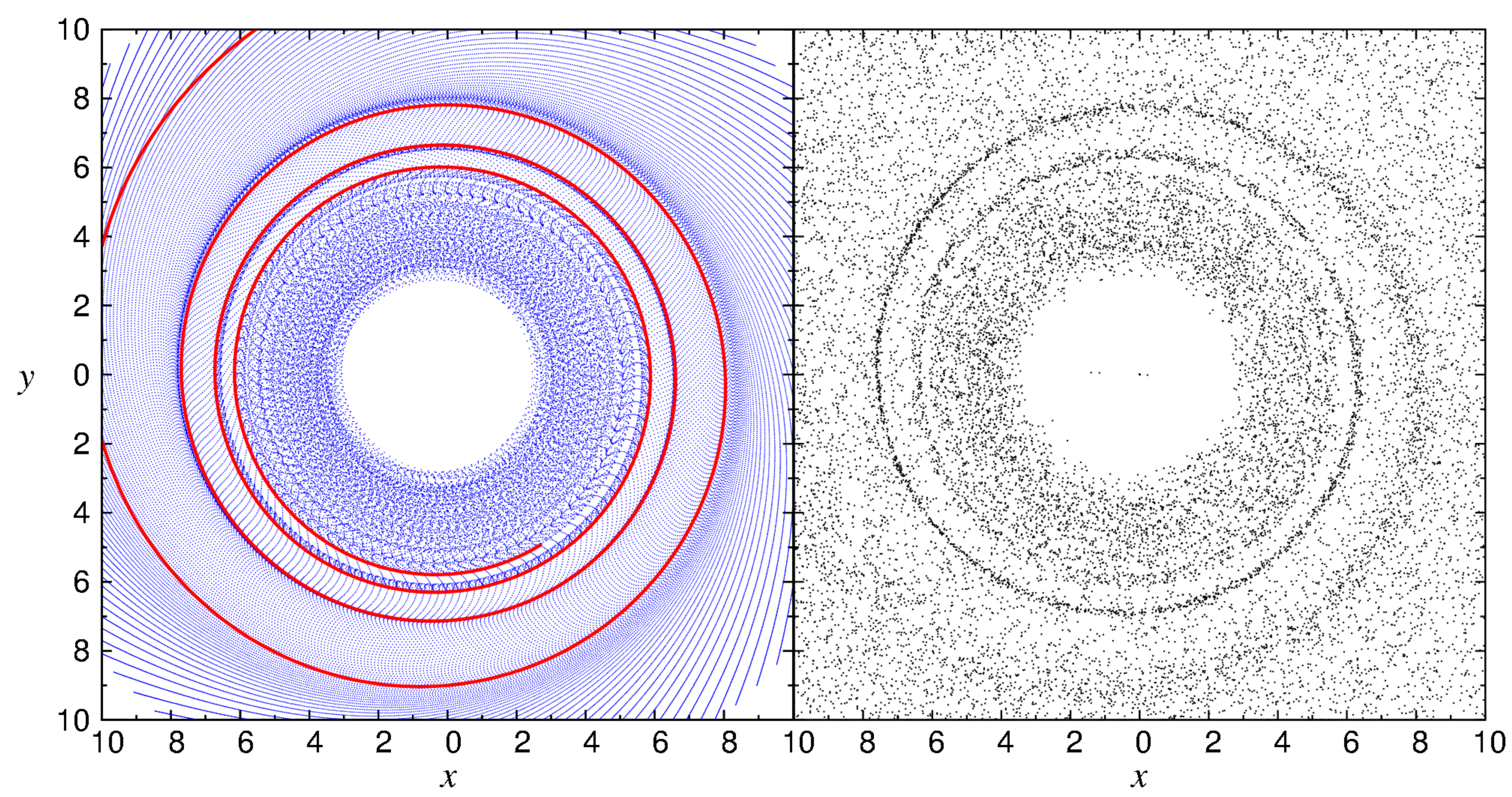

Figure 3: The structure of a stirred planetesimal disk. Left: the disk, given by the secular theory, with the analytical spiral (thick red curve) superimposed. Right: the corresponding SPH simulation.

The analytical spiral is superimposed, revealing close resemblance to the density wave.

References

[1] K. Moriwaki, Y. Nakagawa, *Astrophys. J.* **609**, 1065 (2004).

[2] T.A. Heppenheimer, *Astron. Astrophys.* **65**, 421 (1978)

[3] L.R. Doyle et al., *Science* **333**, 1602 (2011).

[4] S. Meschiari, *Astrophys. J.* **752**, 71 (2012).

[5] S.-J. Paardekooper et al., *Astrophys. J.* **754**, L16 (2012).

[6] G.C.K. Leung, M.H. Lee, *Astrophys. J.* **763**, 107 (2013).

[7] S. Meschiari, eprint arXiv: 1309.4679 (2013).

[8] D.V. Bisikalo *et al.*, *Astron. Rep.* **48**, 449 (2004).

[9] M.C. Wyatt, *Astron. Astrophys.* **440**, 937 (2005).