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Vertical shear instability with radiation transport

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Turbulence in discs

The magneto-rotational instability (MRI) is a strong candidate for sustained turbulence in protoplanetary discs. But it is also known that there may be dead zone somewhere between 2-10 au, where the MRI can only produce very weak turbulence. Recent simulations which included also ambipolar diffusion have even shown no turbulence at all in this region. Thus another instability inside the dead zones is warranted.

Introduction

Recently, the vertical shear (or Goldreich-Schubert-Fricke, GSF) instability was analysed by Nelson et al. (2013) as possible candidate for hydrodynamical turbulence in accretion discs. For isothermal discs they found a weak angular momentum transport with $\alpha = 6 \cdot 10^{-4}$. Because heat is transported from the middle of the disc to the outer parts, the instability dies out. Here, we evaluate the evolution of the instability for radiative discs using an ideal equation of state.

Models

We begin with a locally isothermal disc model in hydrodynamic equilibrium, which is unstable under the GSF-Instability.

$$\begin{aligned}
\rho &= \rho_0 \left(\frac{R}{R_0}\right)^p e^{-\frac{R^2}{H^2}\left(\frac{R}{r}-1\right)} \\
T &= T_0 R^q \\
\Omega &= \Omega_K \left[(p+q)\left(\frac{H}{R}\right)^2 + (1+q) - \frac{qR}{r}\right]
\end{aligned}$$

Here R and Z are polar coordiantes and $r = \sqrt{R^2+Z^2}$. We use $T_0 = 600\text{K}$, $\rho_0 = 10^{-10}\frac{g}{cm^3}$, $R_0 = 1\text{au}$, $p = 1.5$ and $q = -1$. To the velocity we add a random perturbation with an amplitude of 1% of the sound speed.
Initially we use an isothermal equation of state, but we switch to an ideal EOS for radiation transport.

Stability

Nelson et al. (2013) derived the growth rate of the instability for a locally isothermal and compressive gas. They found approximately:

$$\sigma^2 \sim q\Omega\frac{H}{R},$$

This implies that the growth rate per local orbit to first oder depends on the temperature gradient as given by q and on the absolute temperature due to H/R.

References

Mignone, A., Bodo, G., Massaglia, S., et al. 2007, ApJS, 170, 228
Nelson, R., Gressel, O., Umurhan, M., 2013, MNRAS, 435, 2610
Stoll, M. H. R., Kley, W., A&A, submitted to A&A, in revision

Isothermal simulations

The cause of the instability is the hight dependence of the angular momentum as can be seen in the formula for the initial angular velocity. The following figures are simulated from $R = $ 2-10 au and show the growth of kinetic energy in figure 1 and the vertical velocity in figure 2. This waves move inward, resulting in oscillations of the gas around the midplane.

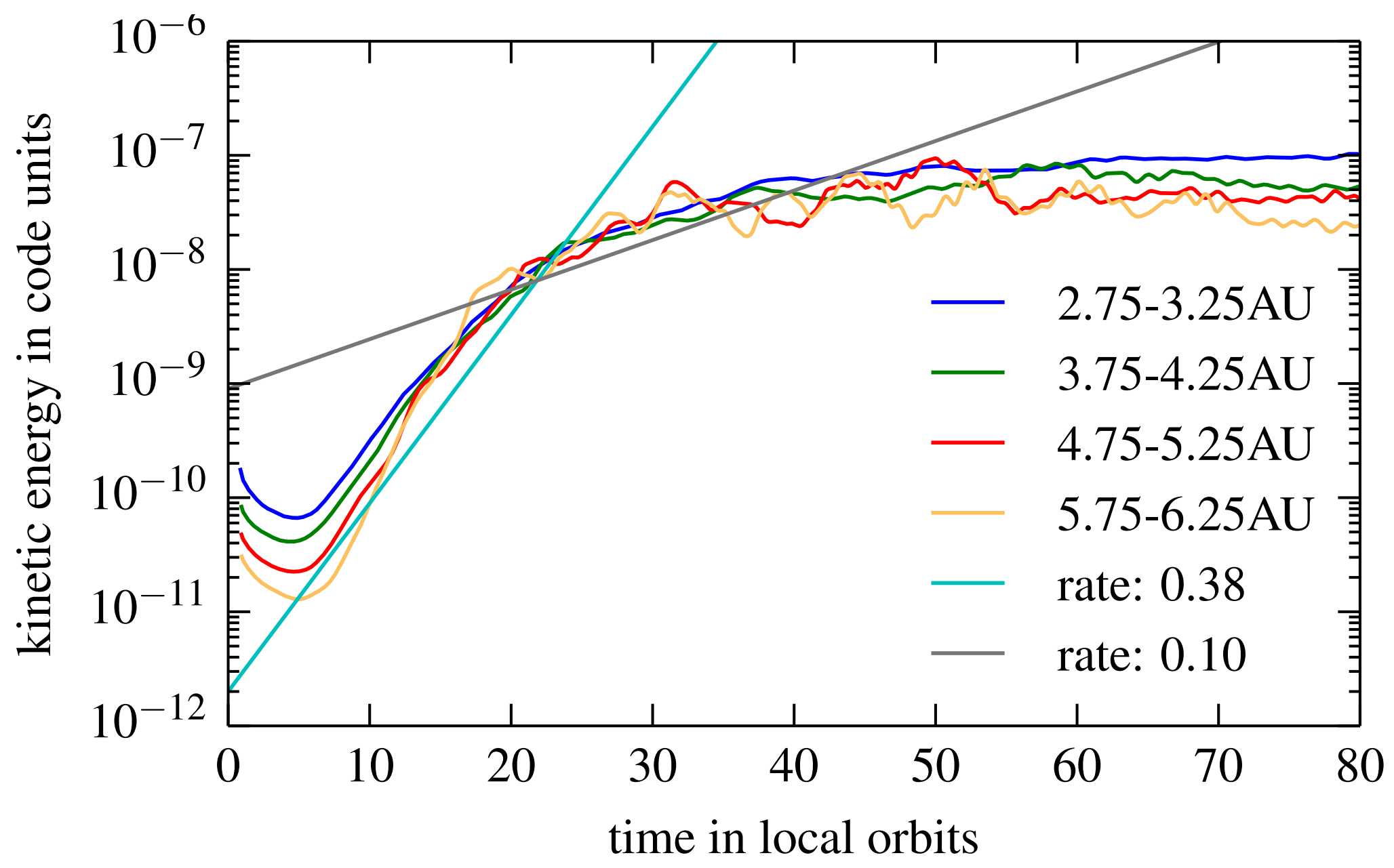

Figure 1: Development of kinetic energy over time. In the first growth phase the instability is symmetric to the midplane, in the second anti-symmetric (see figure 2).

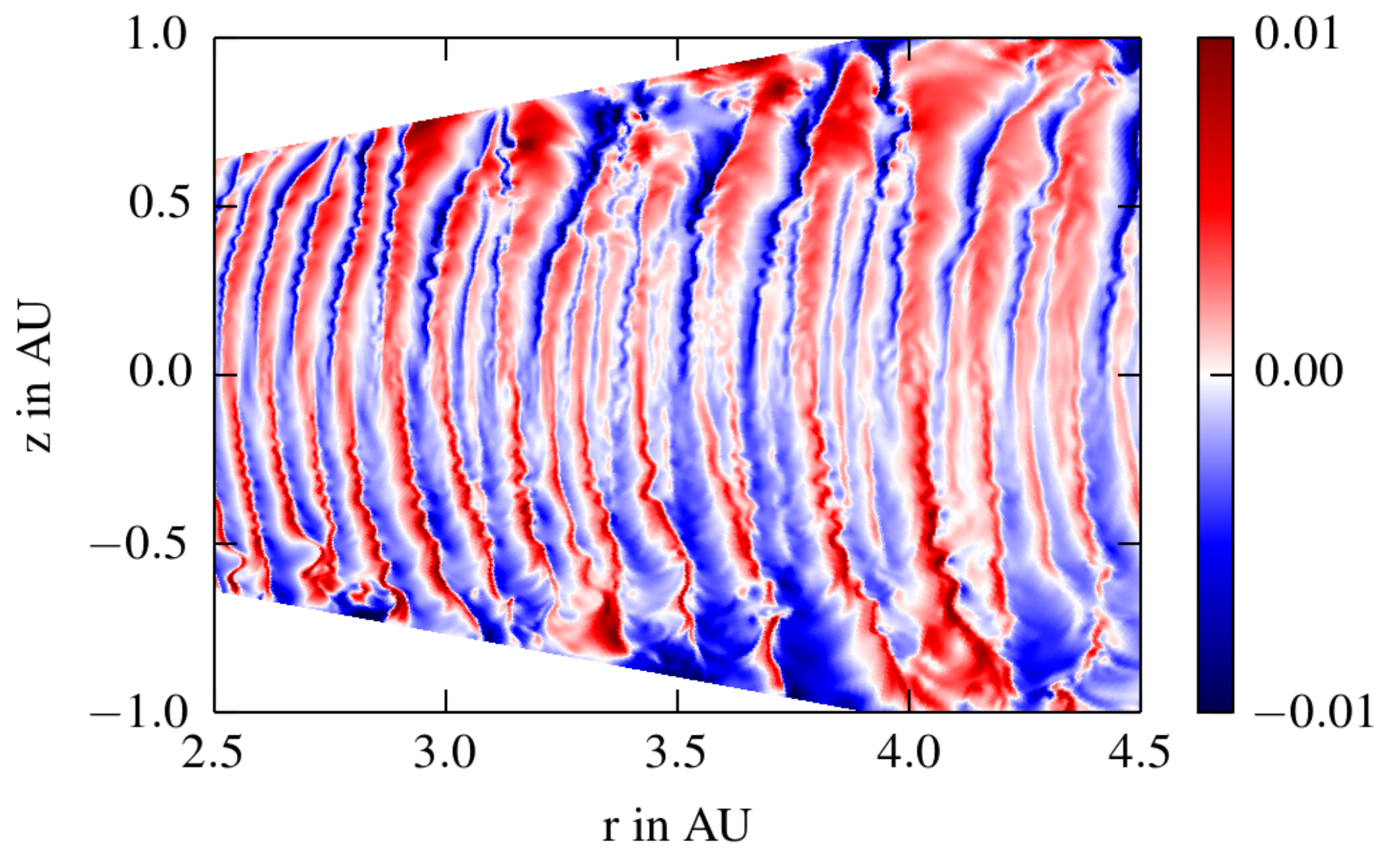

Figure 2: Vertical velocity of a disc with active vertical shear instability for the antisymmetric mode.

Radial wavelength

For a simulation with larger radial extend from 2 - 50 au, we take a closer look at the characteristic wavelength of the instability in the saturated phase, which was expected to be proportional to R.

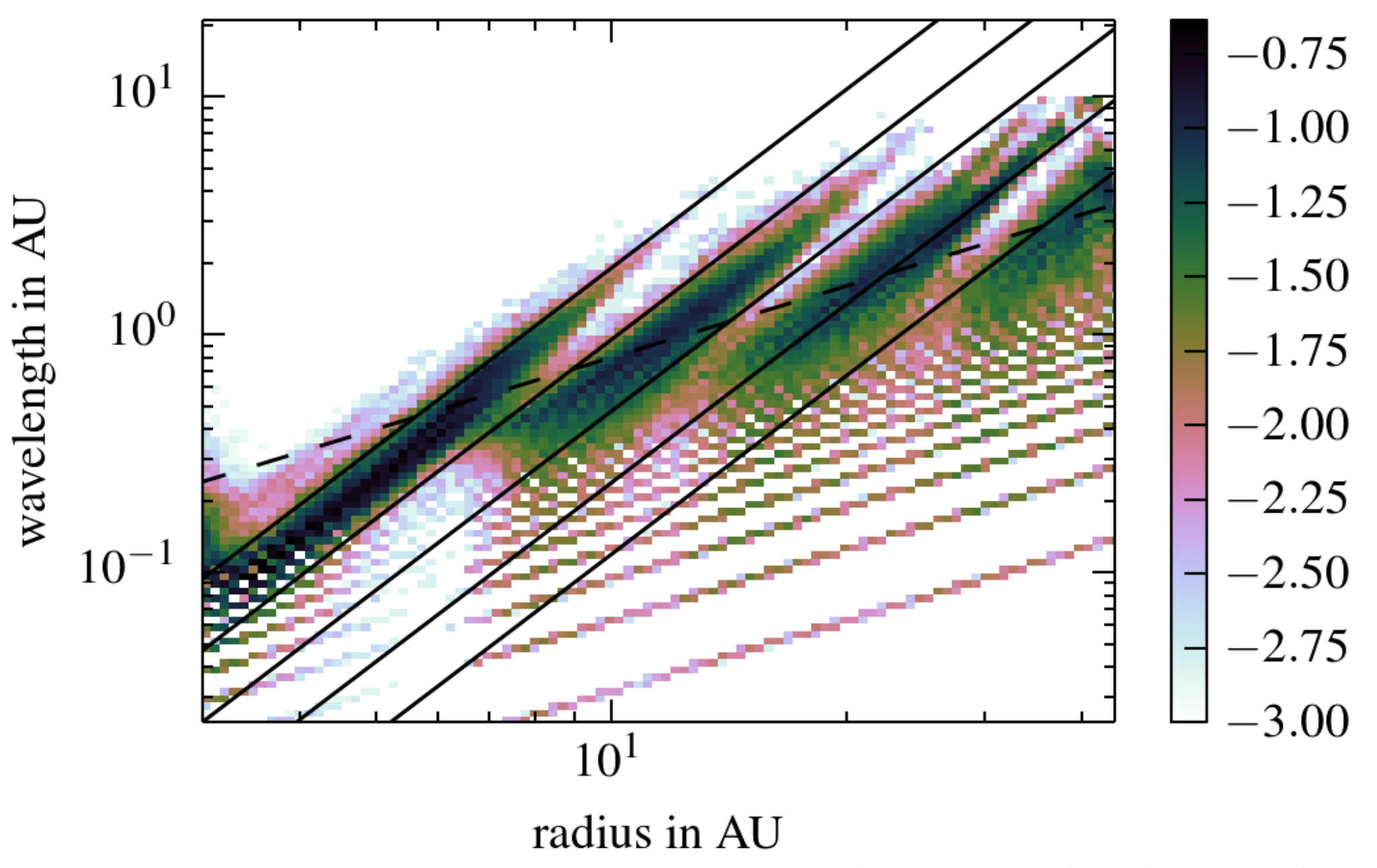

Figure 3: Histogram: Color coded is the logarithm of the probability for the occurrence of a wavelength at a radius normalised at each radius by the sum of all wavelengths for the specific radius. The black lines are proportional to the radius to the power of 2.5 and the lines are a factor of 2 apart from each other. The dashed line has linear slope.

One can see that the instability jumps successively between different modes with $\lambda \sim R^{5/2}$ for the wavelength with corresponding jumps in frequency at the same radius. We assume this unexpected radial wavelength dependence keeps the frequency on every mode independent from radius.

Irradiated Discs

We now add radiation transport, which allows the disc to cool. Due to the low level of turbulence the instability can no longer sustain its temperature profile and cools rapidly. To counter the strong heat loss through radiation transport, we included irradiation from the star. The radiation flux $F = I_* \left(\frac{R_*}{r}\right)^3$, which mimics a flat disc irradiated by a star, is radiated vertically into the disc from both sides. We now find a stable Reynolds stress with $\alpha = 2 \cdot 10^{-4}$, as can be seen in figure 4.

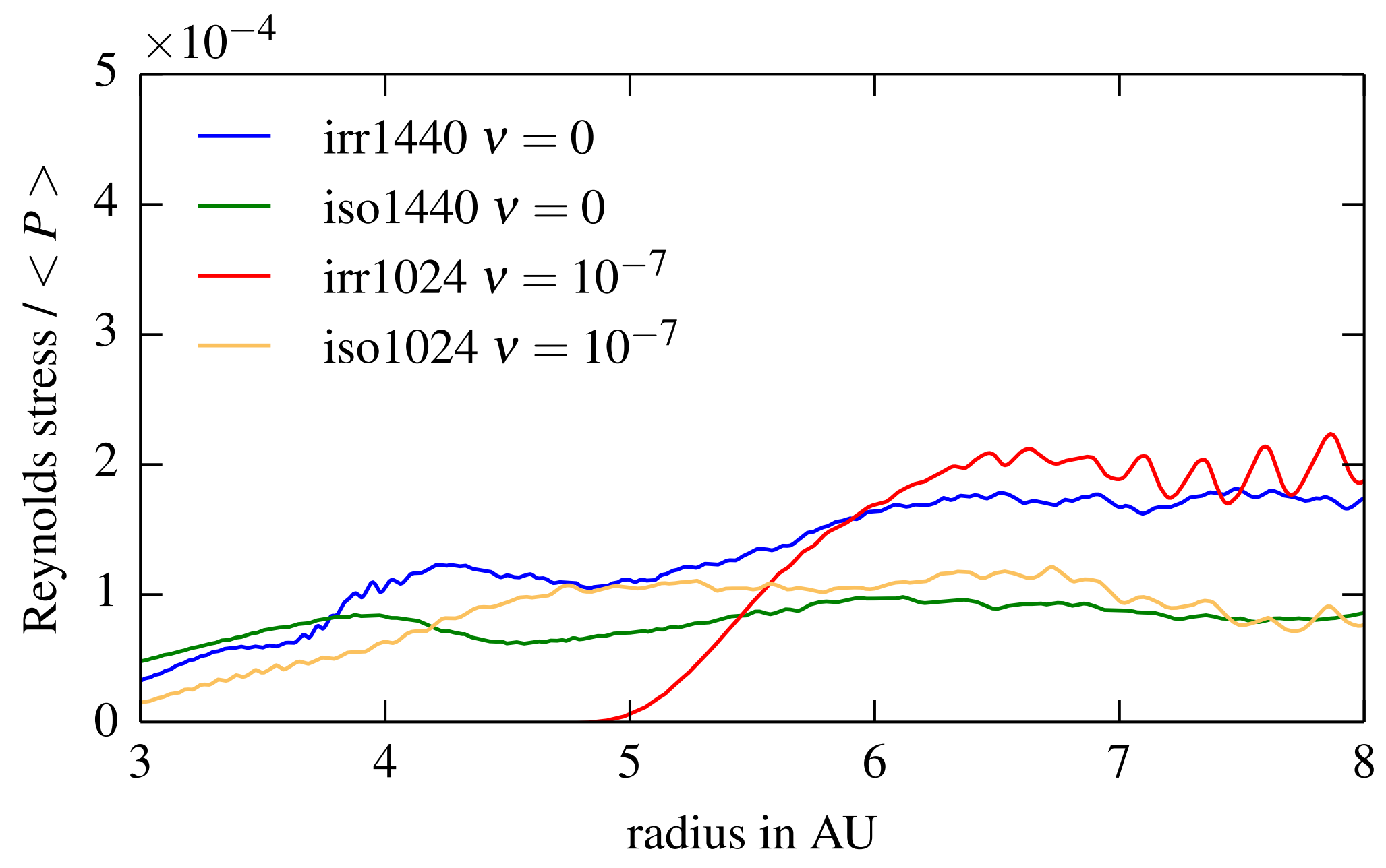

Figure 4: Comparison of time smoothed and space averaged Reynoldsstress/$<P>$ between isothermal and irradiated disc.

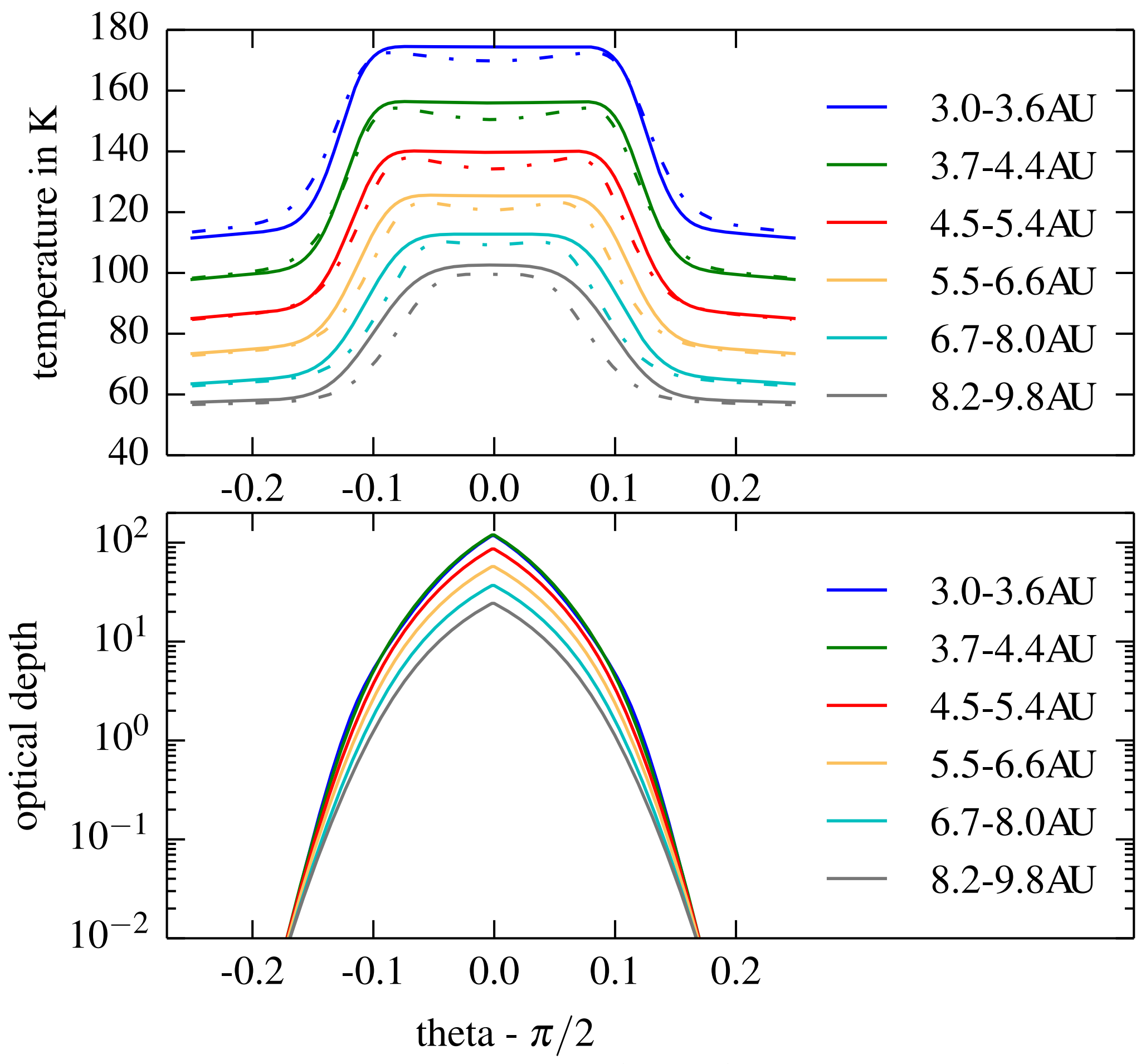

Figure 5: Upper Panel: The temperature profile for the irradiated disc, the dotted line is a run without hydrodynamics. Lower Panel: The vertically integrated optical depth

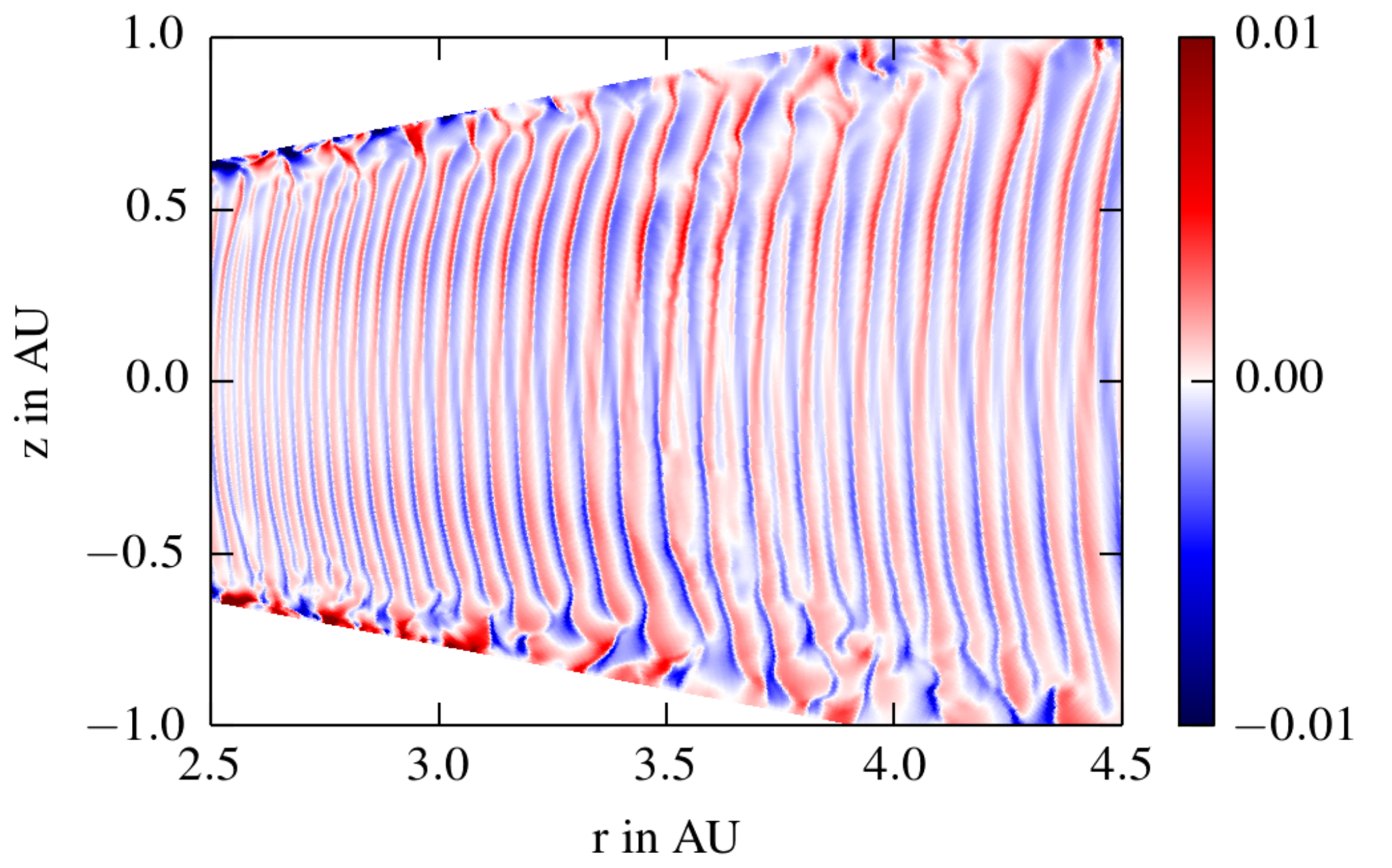

Figure 6: Vertical velocity of a irradiated disc. At 3.5 au the jump in the wavelength is visible.

Summary

Our simulations show that even with radiation transport it is possible for the vertical shear instability to produce a stable turbulence with $\alpha = 2 \cdot 10^{-4}$. This level of turbulence depends on an external source of heat and can also be suppressed by an increase in density, since this leads to large optical depths, preventing the disc to keep temperature profile needed for the instability.