Nonlinear Time Series Analysis of Hyperion's Lightcurves

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Aim

This work is focused on estimating the mLE based on current photometric observations and comparison of the results with the interpretation of other authors. Moreover, conditions that future observations should meet in order to reliably estimate the mLE, are discussed.

Introduction

Saturn's seventh moon, Hyperion, was discovered in the XIX century by Bond [2] and Lassel [9], but it took more than 100 years to obtain its images due to Voyager 2 [13] and Cassini [17] missions. Its shape is highly elongated ($360\times266\times205$ km), making it the biggest known highly aspherical celestial body in the Solar System. Wisdom et al. [18] predicted Hyperion to remain in a chaotic rotational state due to its high oblateness. In dynamical system theory, a chaotic behaviour is recognised through a positive maximal Lyapunov Exponent (mLE), which describes the rate of divergence (or convergence in the negative case) of initially nearby phase-space trajectories. The Lyapunov spectrum is relatively easy to calculate in the case when the differential equations are known [1, 3, 11, 12, 19]. On the other hand, there exist algorithms allowing to obtain a mLE from an experimental or observational time-series [6, 19], although they are to be used with carefullness, using at least a few hundred data points [7, 14]. It is a hard task in astronomy to obtain long-term, well-sampled lightcurves. Although, despite this difficulty, it has been efficiently shown that pulsar spin-down rates exhibit chaotic dynamics [15]. By re-sampling the original measurements, artificial time-series were produced, equivalent to the original ones, containing such a number of data points that the calculation of the correlation dimension and the mLE of the attractor, reconstructed *via* Takens time delay embedding method [16], was possible.

Figure 1: Hyperion as viewed by Cassini.

Datasets

Hyperion's long-term observations were carried out twice in the post Voyager 2 era. In 1987, Klavetter [8] (hereinafter, K89) performed photometric *R* band observations over a time-span of more than 50 days, resulting in 38 high-quality data points. In 1999 and 2000, Devyatkin et al. [4] (hereinafter, D02) conducted *C* (integral), *B*, *V* and *R* band observations. According to the author's knowledge (Mel'nikov, private communication) there were no other long-term observations that resulted in a lightcurve allowing to determine the rotational state of Hyperion. Although, shortly after the Cassini 2005 passage a ground-based *BVR* photometry was conducted [5], resulting in 6 nights of *BVR* measurements (and additional 3 nights of *R* photometry) over a month-long period. Unfortunately, this data was undersampled and period fitting procedures yielded several plausible solutions.

Results of processing the observations

The programmes described in [10, 21] have been used to process the photometric and simulated lightcurves. The mutual information and false nearest neighbours have been used to compute the embedding parameters.

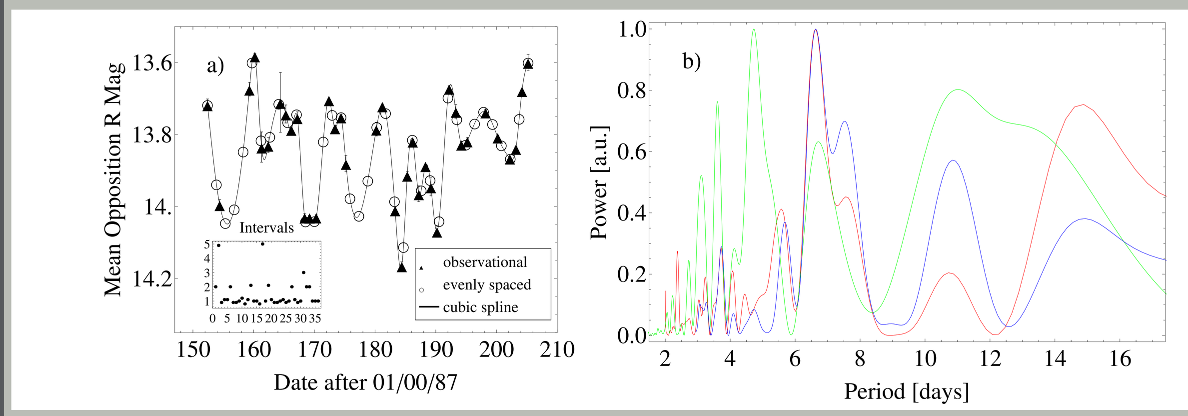

Figure 2: a) The original K89 data (triangles) and the equally spaced re-sampled ones (circles). The interpolation was performed using a natural cubic spline, which was next sampled to form a 5000 points equally spaced dataset. b) Lomb-Scargle periodogram of the K89 lightcurve: red – original data, green – sampled with a time step equal to the mean step of the original data, blue – sampled with a time step short enough to form a 5000 points time-series. The vertical axis is in normalised auxiliary units.

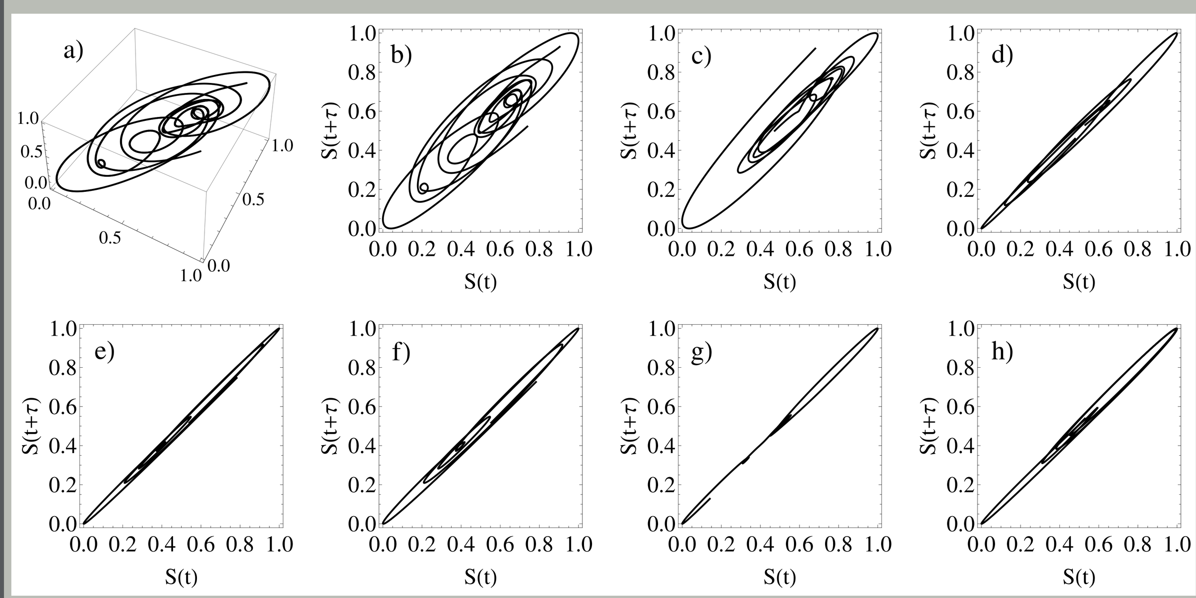

Figure 3: Phase space trajectories reconstructed using the Takens delay time method. All embeddings appear to posses the same topology, indicating each trajectory stems from the same underlying dynamics. The corresponding datasets are: a) K89 in 3D b) K89 in 2D, c) C1, d) C2, e) R1, f) R2, g) B2, h) V2. Note that all of the reconstructions posses the same topology, although d) – h) look like being of a purely regular time-series. This may be due to the undersampling of their corresponding lightcurves. The correlation dimensions for these structures are b) 1.31, c) 1.18, d) 1.20, e) 1.25, f) 1.26, g) 1.13, h) 1.05.

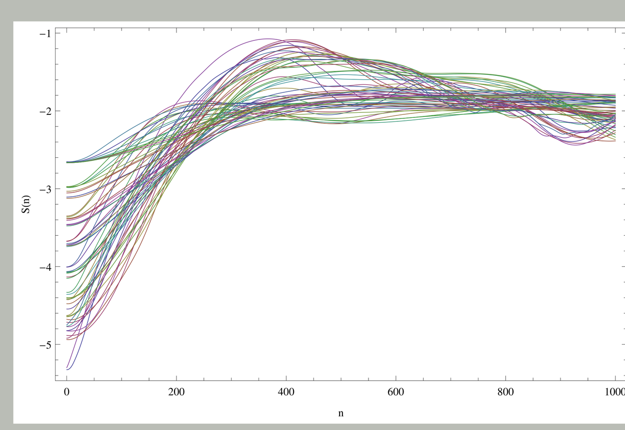

Figure 4: The Kantz algorithm applied to K89 data. No unambiguous linear part is recognizable, hence no positive mLE can be assigned to this lightcurve. The data from D02 all possess the same feature. The observations are either too short, or undersampled, or both, to obtain a reliable estimate of the mLE.

Results of numerical simulations

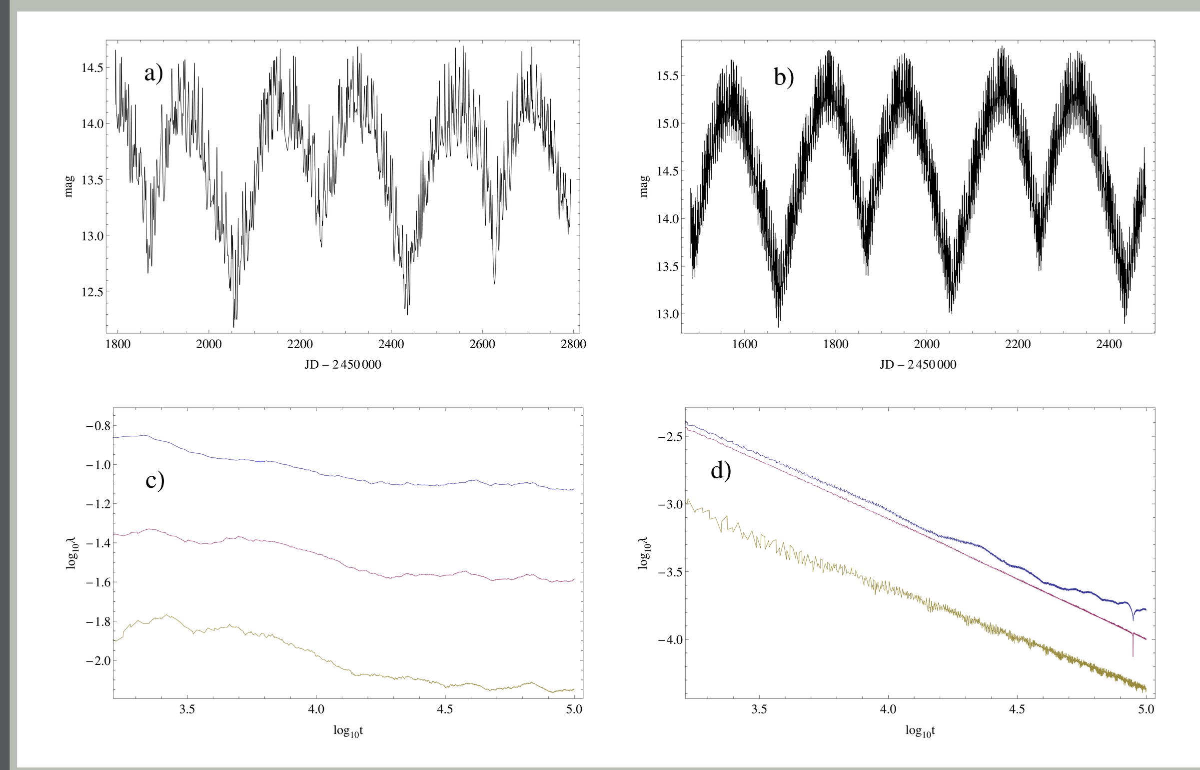

Figure 5: Simulated lightcurves for the a) chaotic and b) regular initial conditions (Mel'nikov, private communication). The dynamical system (Euler equations) is Hamiltonian and six dimensional, therefore there are pairs $\lambda_i = -\lambda_j$. The three independent LEs c) plateu to constant, positive values (0.077233, 0.025985, 0.007203) for the chaotic case and d) decrease linearly for the regular region [20].

The lightcurves in Figs. 5 a) and b) have a constant $\Delta t = 0.1$ d, so in order to produce more realistic time-series only first three points in each day were left and averaged. Then a cubic spline and the sampling were performed to produce data consisted of 5000 points. From these sampled lightcurves, intervals of lengths of 2 months, 6 months and 1 year were chosen randomly; each had ten realisations. The whole 3 year lightcurve was taken as one realisation.

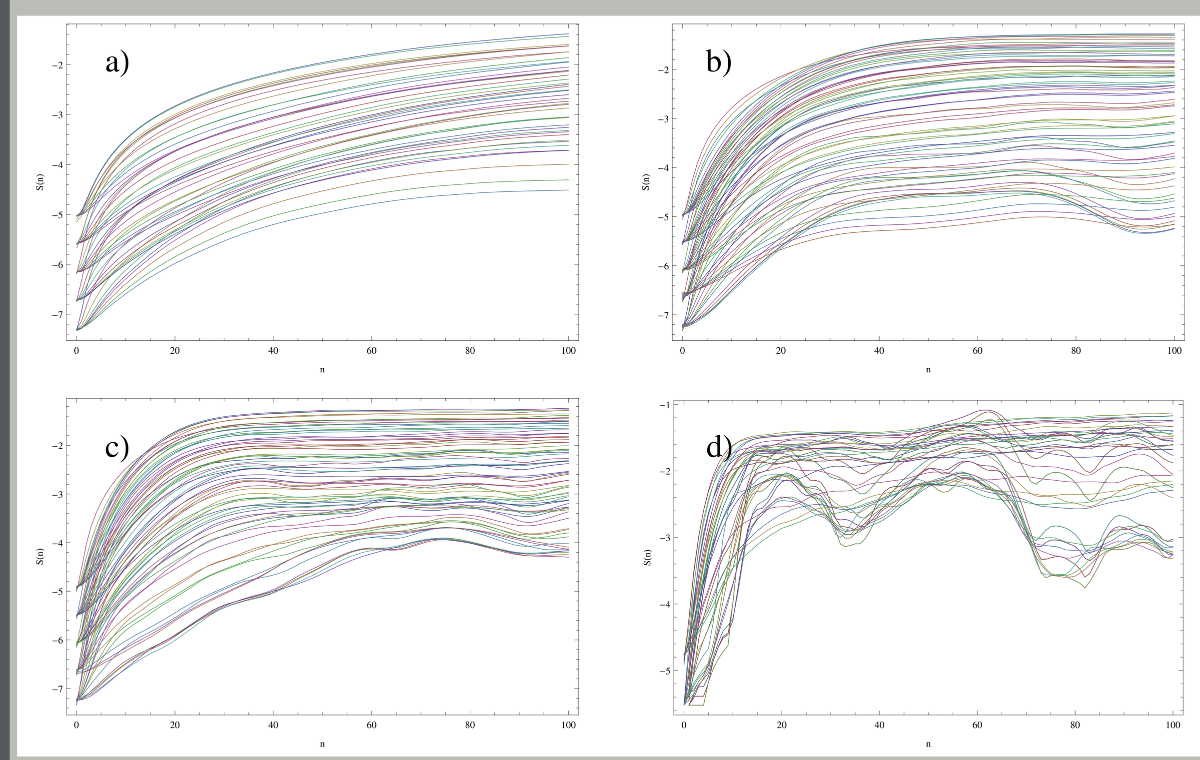

Figure 6: Examples of Kantz algorithm applied to the simulated lightcurves of lengths a) 2 months, b) 6 months, c) 1 year and d) 3 years.

Conclusions and future work

In this work an attempt to estimate the mLE from photometrically obtained lightcurves of Hyperion has been made. The existing astronomical data (K89 and D02) are either too short, or undersampled to conduct the proper processing. Intervals of various length were extracted from the simulated lightcurves spanning $\approx$ 3 years. The 2 months-long data are too short for the positive mLE to be visible. Yet, the lightcurves spanning periods of 6 months and longer exhibit a linear part in the logarithm of the stretching factor, which is a confirmation of a chaotic rotation. Therefore, one needs to perform longer and well-sampled observations for a reliable estimation of the mLE. The next step will be estimating the value of the mLE and checking how it is influenced by the sparseness of data.

Acknowledgements

The author is grateful to Andrew Seymour for discussions and Alexander Mel'nikov for providing the simulated lighcurves as well as useful information.

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