

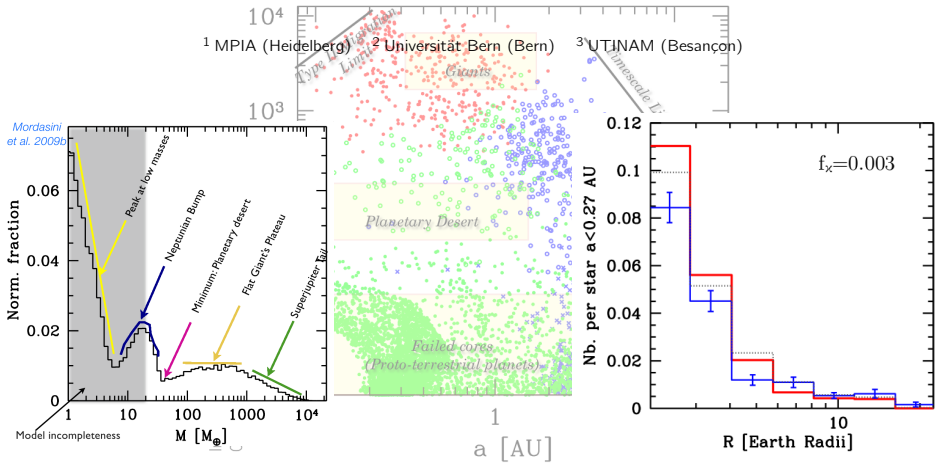
Planetary population synthesis

Statistical tests of and constraints on planet formation

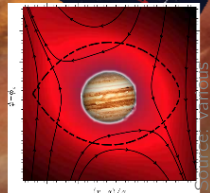
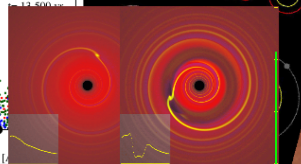
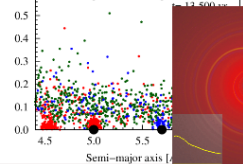
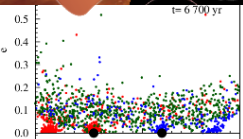
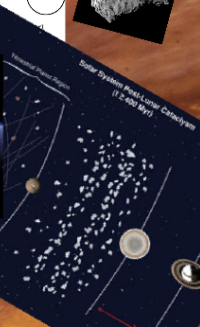
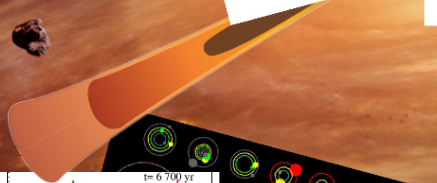
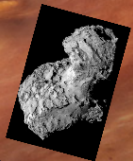
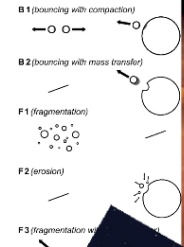
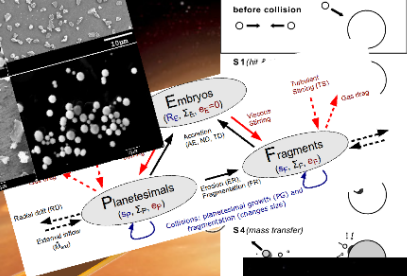
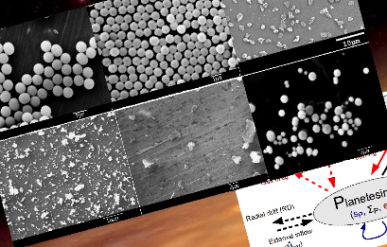
G.-D. Marleau¹

C. Mordasini¹, P. Mollière¹, H. Klahr¹, Th. Henning¹

A. Fortier², D. Swoboda², Y. Alibert^{2,3}, W. Benz²



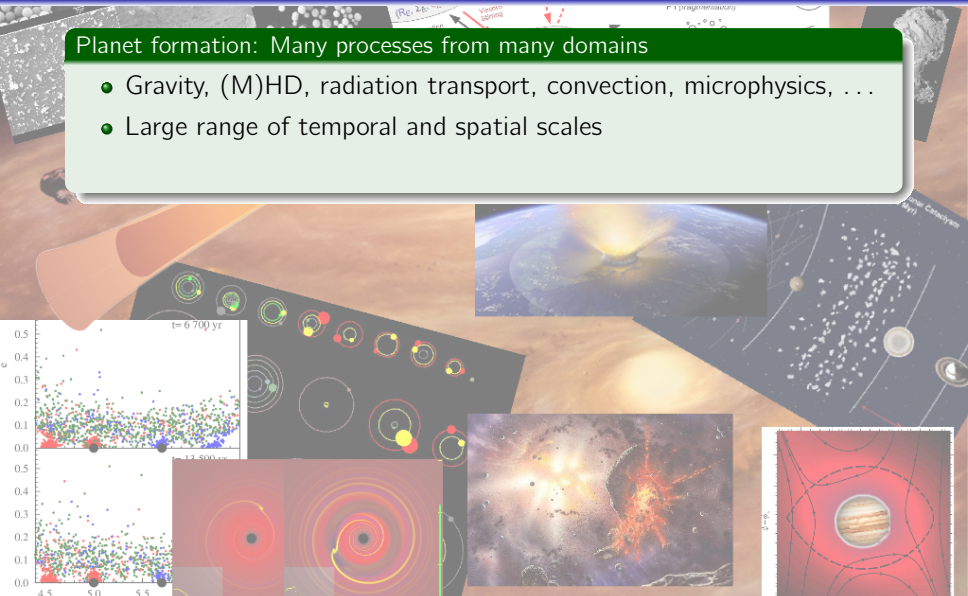
Mordasini
et al. 2009b



The essence of population synthesis

Planet formation: Many processes from many domains

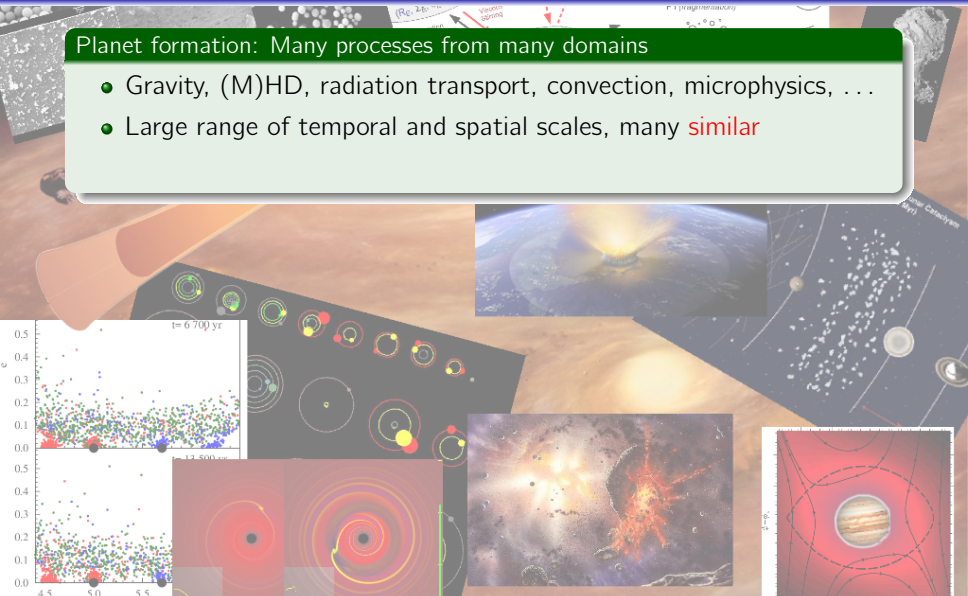
- Gravity, (M)HD, radiation transport, convection, microphysics, ...
- Large range of temporal and spatial scales



The essence of population synthesis

Planet formation: Many processes from many domains

- Gravity, (M)HD, radiation transport, convection, microphysics, ...
- Large range of temporal and spatial scales, many **similar**

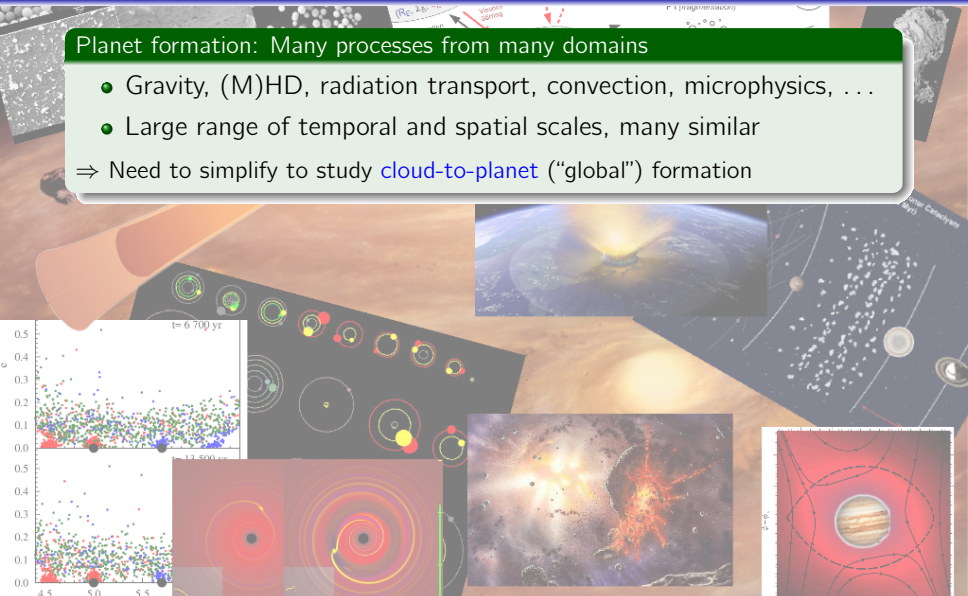


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⇒ Need to simplify to study **cloud-to-planet** (“global”) formation

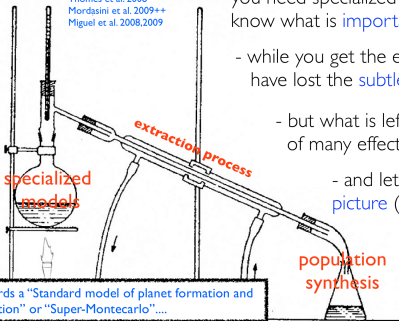


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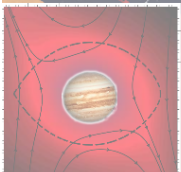
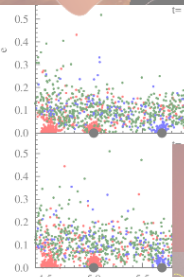
Ida & Lin 2004++
Thomes et al. 2008
Mordasini et al. 2009++
Miguel et al. 2008, 2009



- you need specialized models to know what is **important**
- while you get the essence, you have lost the **subtlety** of the original
- but what is left is a **concentrate** of many effects
- and lets you see the **big picture** (hopefully)

Towards a "Standard model of planet formation and evolution" or "Super-Montecarlo"....

W. Benz

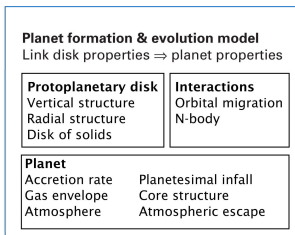


Schematic principle of population synthesis

Diversity of planets: same physics + diversity of initial conditions

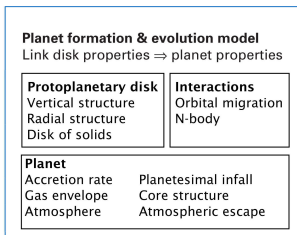
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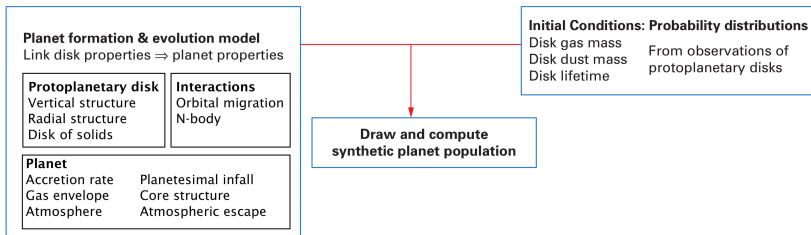


Initial Conditions: Probability distributions

Disk gas mass	From observations of protoplanetary disks
Disk dust mass	
Disk lifetime	

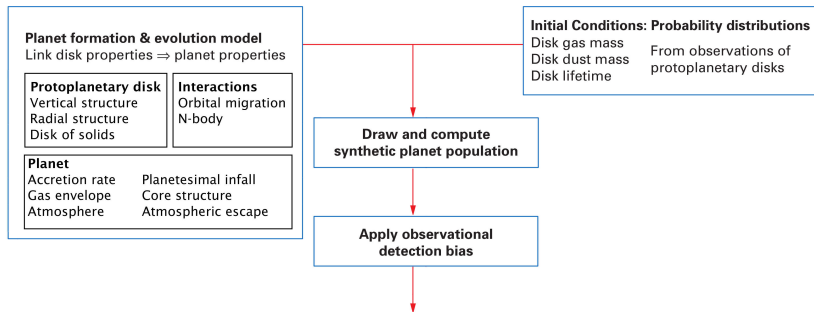
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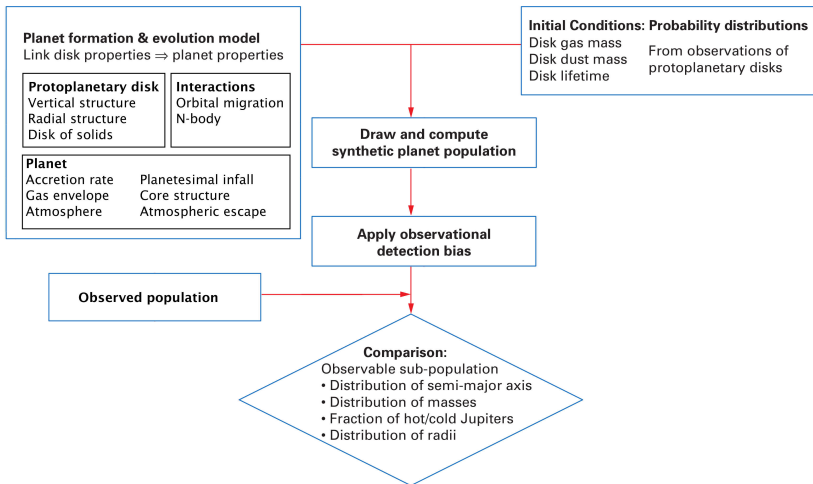
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Schematic principle of population synthesis

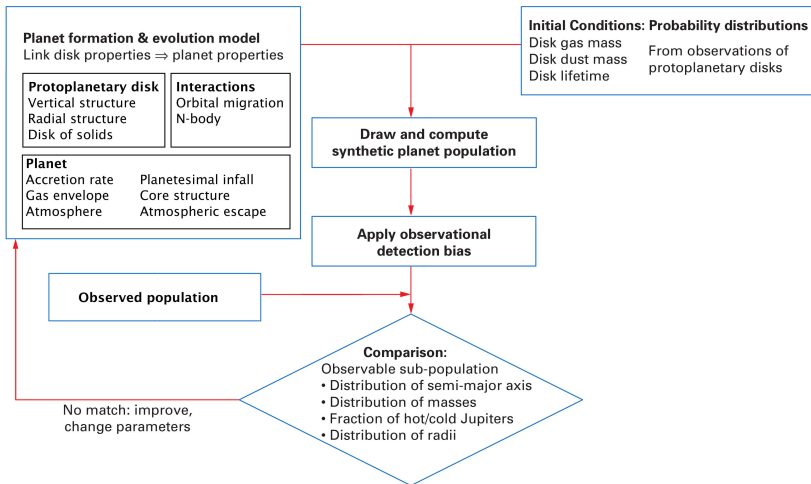
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Mordasini et al. (2014c)

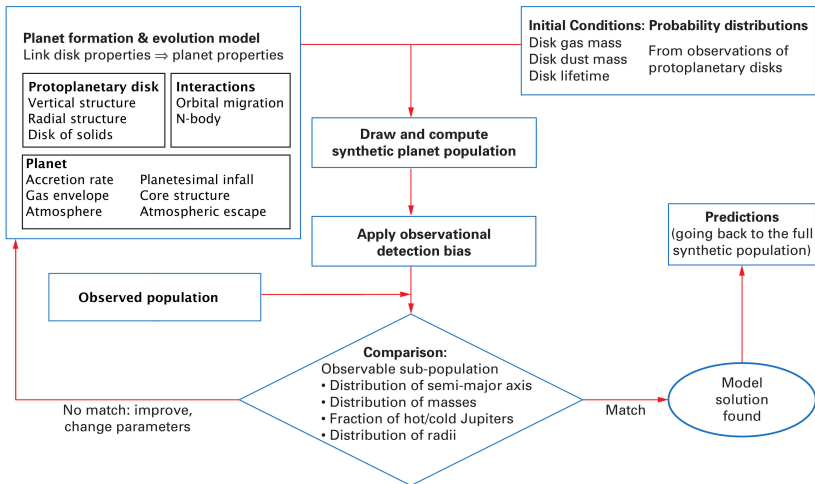
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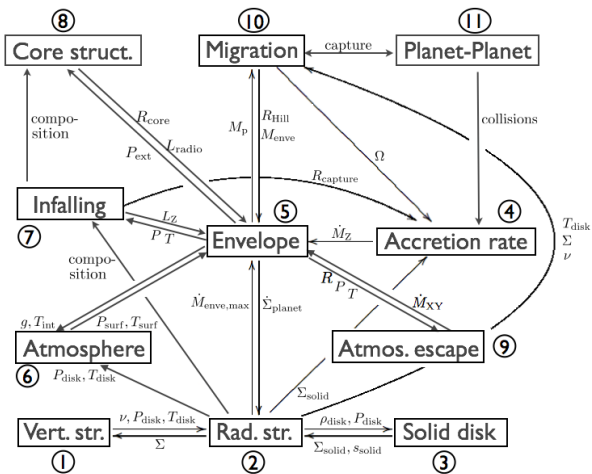


Schematic principle of population synthesis

Diversity of planets: same physics + diversity of initial conditions



The main modules



(Alibert et al. 2005, Fouchet et al. 2011, Mordasini et al. 2012b, Alibert et al. 2013, Fortier et al. 2013, Dittkrist et al. 2014, Jin et al. 2014, Marleau et al. in prep.)

(Incomplete!) relevant literature

Pioneering and recent work:

- ★ Ida & Lin et al. (2004–. . .). *Toward a deterministic formation model. I–VII*
- ★ Alibert, Mordasini, Benz & Winisdoerffer (2005). *Models of giant planet formation with migration and disc evolution*

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Reviews:

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This talk

Focus on [core accretion](#)

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Overview

- 1 Selected physical ingredients
 - Disc-related modules
 - Planet-related modules
- 2 Comparisons to observations: constraints
 - General comments
 - Mass–semi-major axis diagram (M_p – a)
 - Mass–radius relationship (M_p – R_p)
- 3 Summary

Overview

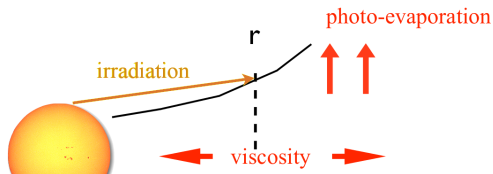
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Disc structure and evolution

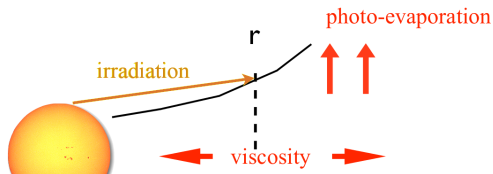
- Standard α -viscosity, 1+1D disc (parametrised/numerical)
- Internal (central star) + external (cluster stars) photoevaporation
- Initial profile: theory/observations



Y. Alibert

Disc structure and evolution

- Standard α -viscosity, 1+1D disc (parametrised/numerical)
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Y. Alibert

Open question: Growth of solids from μm to $\sim \text{km}$

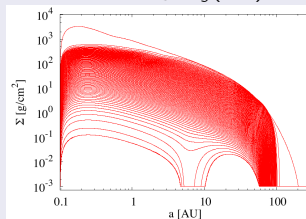
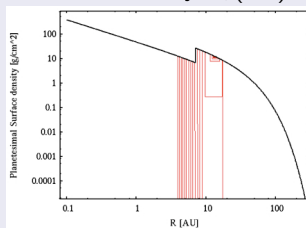
- Two-body coagulation/self-gravity, high τ_{runaway} dependence on turbulence, ...
 - Start with bimodal: $0.6 M_{\oplus}$ protoplanet + 100-km planetesimals
- ⇒ Need more results from specialised models

(Birnstiel et al. 2010, Ormel & Okuzumi 2013, Johansen et al. 2009, 2014, etc.)

Disc structure and evolution

- Standard α -viscosity
- Internal (centrifugal) migration
- Initial profile:

Numerical model

Gas surface density $\Sigma_g(r, t)$:Solids surface density $\Sigma_d(r, t)$:

numerical)
photoevaporation

on

Y. Alibert

on turbulence, ...
imals

sen et al. 2009, 2014, etc.)

Open question: Growth

- Two-body coagulation
 - Start with bimodal size distribution
- ⇒ Need more results

Orbital migration

Planet–disk interaction → migration + e, i damping

- 2/3D rad.-hydro simulations expensive
- Migration rates: simulations / theory

(e.g. Kley & Nelson 2012, Dittkrist et al. 2014, Paardekooper et al. 2011, 2014)

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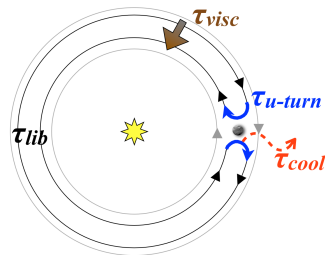
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Orbital migration

Planet–disk interaction → migration + e, i damping

- 2/3D rad.-hydro simulations expensive
 - Migration rates: simulations / theory
 - Isothermal → too fast Type-I rate
- ⇒ Motivated theoretical developments
- Find: thermal structure is crucial
 - ★ Compare timescales → which regime



Dittkrist et al. (2014)

(e.g. Kley & Nelson 2012, Dittkrist et al. 2014, Paardekooper et al. 2011, 2014)

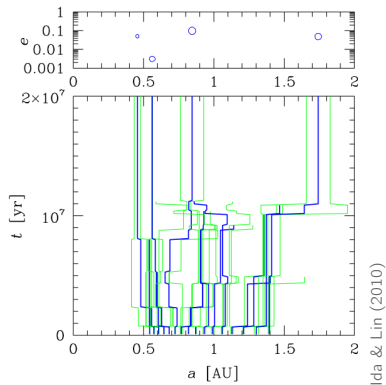
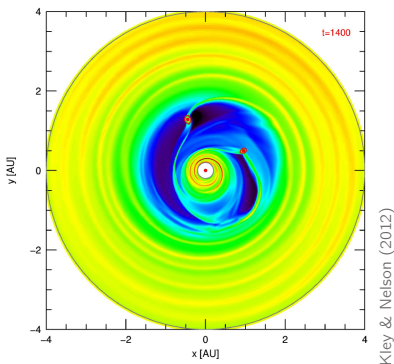
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Planet-planet interactions

- Effects: competitive accretion, MMR, collisions, ejections, ...
- Computation: explicit N -body / statistical (timescales) approach

(Ida & Lin 2010, Ida et al. 2013, Alibert et al. 2013)



Planetary structure

Gas accretion rate: $\min(\dot{M}_{\text{thermal}}, \dot{M}_{\text{disc}})$

- Semi-analytical approach (Ida & Lin): $\dot{M}_{\text{thermal}} \simeq M_p / \tau_{\text{KH}} \propto M_p^{4-5}$
- Solution of standard structure equations (Alibert, Benz, Mordasini)

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Key for structure and \dot{M}_{thermal} : **atmospheric opacity** $\kappa_{\text{atm}} \ll \kappa_{\text{ISM}}$

- Detailed grain evolution model

(e.g. Podolak 2003, Movshovitz et al. 2010)

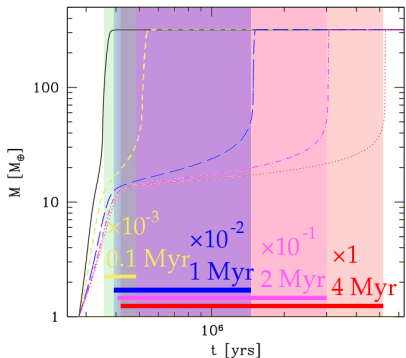
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 - Classic, arbitrary: $f_{\kappa} = 2\%$
 - $f_{\kappa} = 0.3\%$ (Mordasini et al. 2014a)



Mordasini et al. 2014a

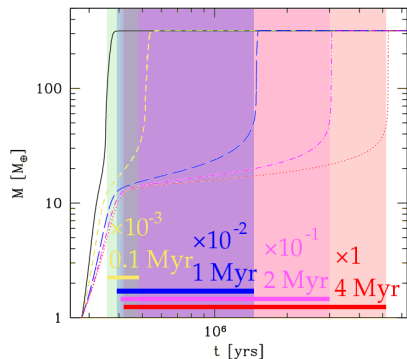
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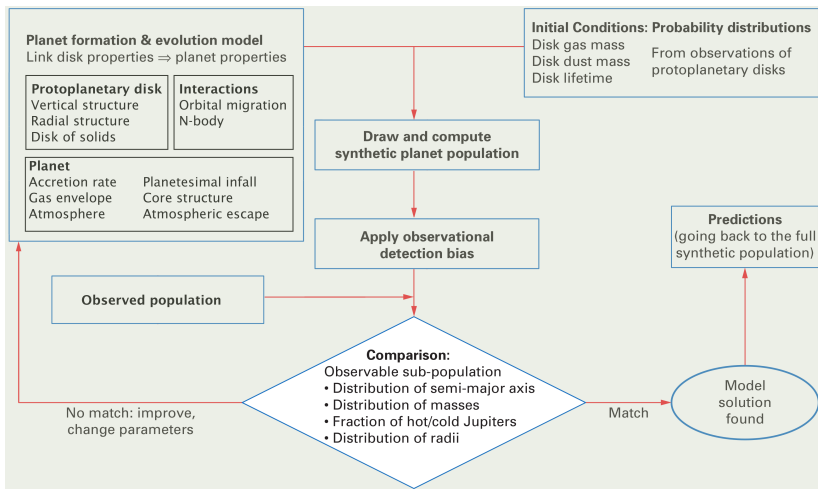
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Comparing population synthesis to data

- Need to **simultaneously** satisfy constraints on:
 - ① Frequencies: overall, by planet type, $f(\text{stellar type})$, $f([\text{Fe}/\text{H}])$
 - ② Planet properties: M_p , R_p , composition, spectrum
 - ③ Syst. architectures: a_p , e , planet type as $f(a_p)$, multiple-planet

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 - Number of parameters \ll number of constraints
 - Small range for some parameters \Rightarrow included **physics** important
- Disagreements show where further work is needed

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Key point

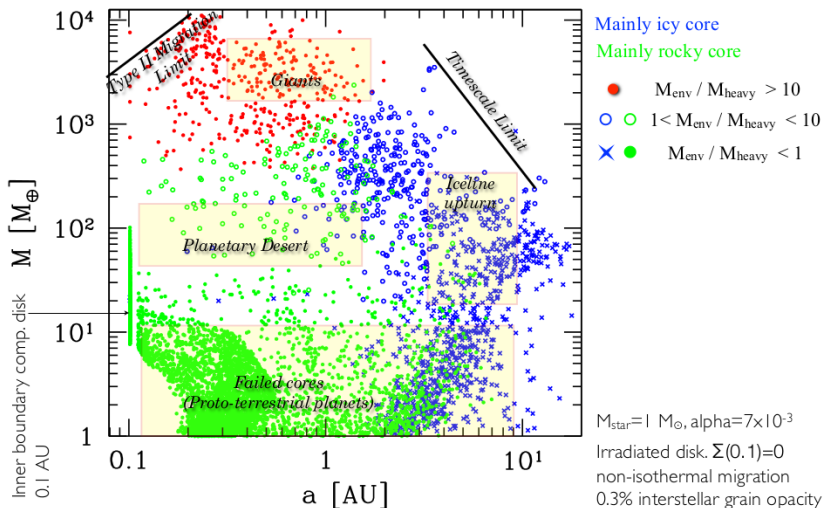
- ★ With population synthesis: can compare **specialised models** to **data!** ★

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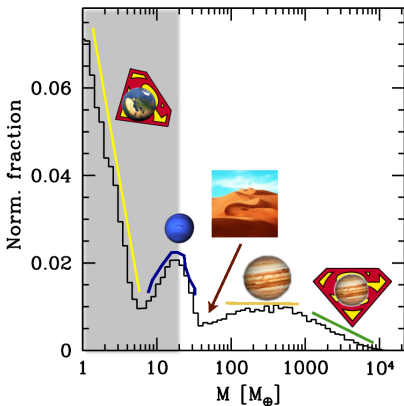
Mass–semi-major axis diagram (M_p – a)

Main features



Planetary Initial Mass Function (PIMF)

Solid + gas accretion rate \rightarrow PIMF

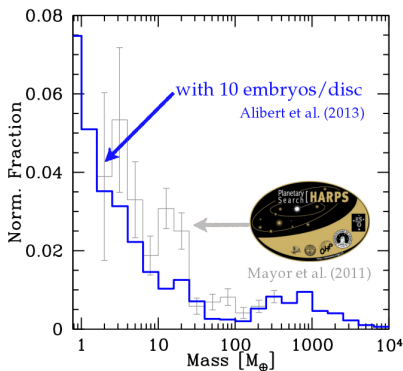


Predictions of CA (features):

- Decreasing $N_{\text{super-jupiters}}(M_p)$
- Gas-giant plateau
- Planet desert at $\approx 30 M_{\oplus}$
- Neptunian bump
- Large rise towards low masses

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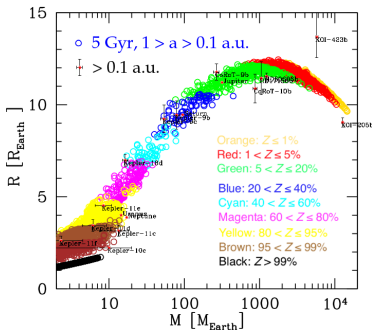
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Mass–radius relationship and composition

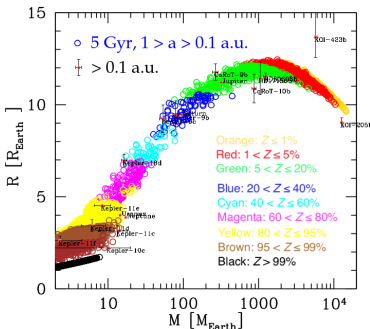
Late-time mass–radius relationship



- Good overall agreement
- Shape: prediction of CA

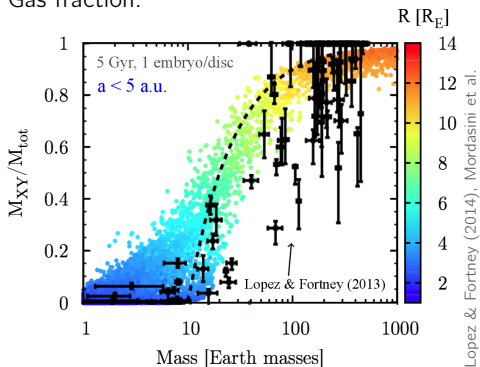
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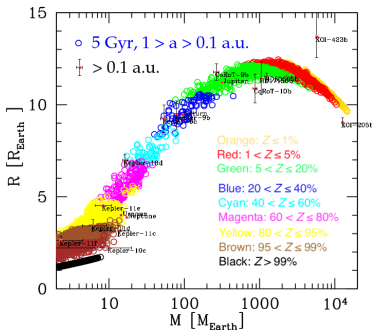
Gas fraction:



- Simulation: ≈ 10 - M_{\oplus} -core line?
- “See” runaway accretion!

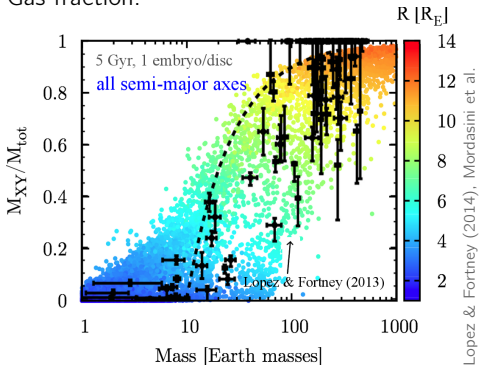
Mass–radius relationship and composition

Late-time mass–radius relationship



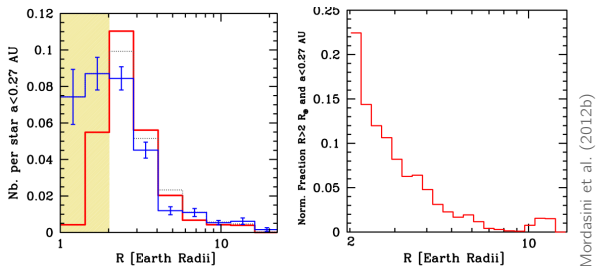
- Good overall agreement
- Shape: prediction of CA

Gas fraction:



- Simulation: ≈ 10 - M_{\oplus} -core line?
- “See” runaway accretion!
- Need planet–planet scattering

Radius distribution

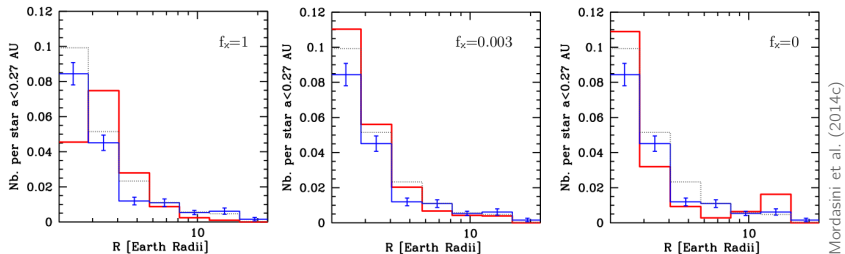


Mordasini et al. (2012b)

$P < 50$ days ($a_p < 0.3 \text{ AU}$ for $1 M_\odot$), *Kepler* data

- Only H/He-atmospheres \rightarrow ignore $< 2 R_p$
- Increase at low M_p + solid-dominated \rightarrow increase at small R_p
- Using smaller bins: see Jovian peak (from flat $M_p(R_p)$ at $3 M_J$)

Radius distribution II



⇒ Constraints on atmospheric opacity

Overview

- 1 Selected physical ingredients
 - Disc-related modules
 - Planet-related modules
- 2 Comparisons to observations: constraints
 - General comments
 - Mass–semi-major axis diagram (M_p – a)
 - Mass–radius relationship (M_p – R_p)
- 3 Summary

Summary

The main power of population synthesis

With population synthesis: can compare specialised models with data

- Type I migration rate
- Grain opacity

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Highlights:

- ★ Predictions for the Planetary Initial Mass Function
 - 1 Change of slope at $\approx 30 M_{\oplus}$
 - 2 Large increase at low masses

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Summary



Thank you for your attention!



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Additional material

- 4 Random variables for core accretion population synthesis
- 5 Planetary structure
- 6 Formation tracks in the mass–semi-major axis plane
- 7 Accretion rate effect

Additional material

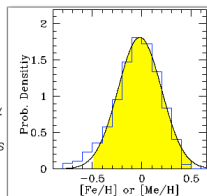
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Initial conditions

1 Metallicity

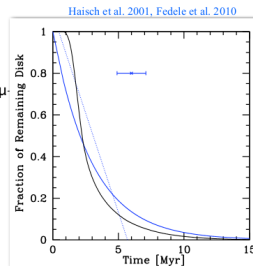
assume same in star
and disk

Stellar $[Fe/H]$ from spectroscopy.
Gaussian distribution for $[Fe/H]$
with $\mu \sim 0.0$, $\sigma \sim 0.2$. (e.g. Santos
et al. 2003)



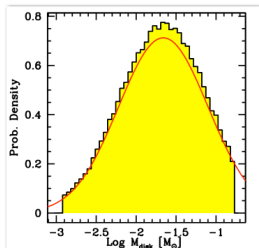
3 Disk lifetime

L-band ($3.4 \mu m$)
photometry:
- excess caused by μ -
sized dust @ $\sim 900K$
... ok to $< 10 AU$



2 Disk (gas) masses

Thermal continuum emission from cold dust at mm
and submm wavelengths (Ophiuchus nebula).



4 Initial semimajor axis of the seed embryo:

Analytical work (Lissauer & Steward 1992) and
numerical simulations (Kokubo & Ida 2000): spacing
between bodies $\Delta \propto a$

$$p(a)da \propto \frac{da}{\Delta} \propto \frac{da}{a} = d\log(a) \propto const.$$

5 Stellar mass

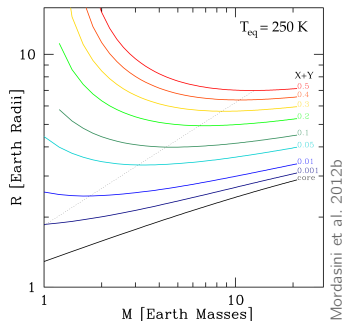
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Planetary structure II

Differentiated **solid core** (Fe & Ni + silicates + ices, from formation)

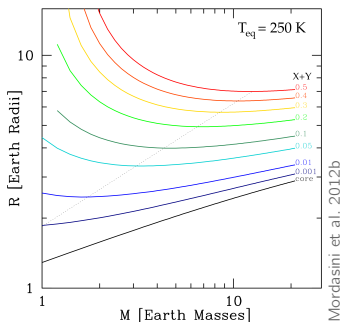
- Modified polytrope: $\rho(P) = \rho_0 + cP^{0.5-0.6}$ (Seager et al. 2007)



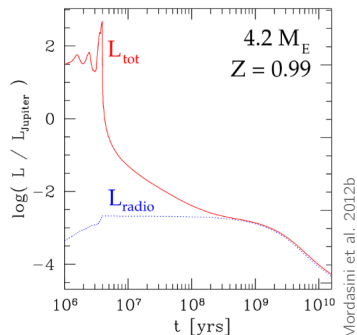
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Differentiated **solid core** (Fe & Ni + silicates + ices, from formation)

- Modified polytrope: $\rho(P) = \rho_0 + cP^{0.5-0.6}$ (Seager et al. 2007)
- Luminosity \rightarrow cooling:
 - Radioactive decay important at late times (and early?)



Mordasini et al. 2012b



Mordasini et al. 2012b

Planetary structure III

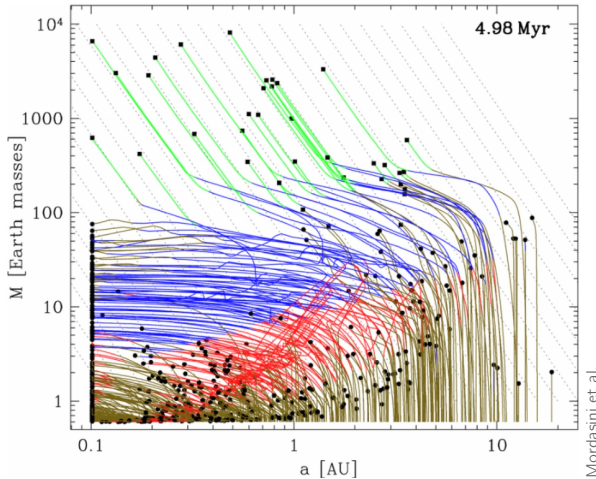
Recent and coming developments:

- Atmospheric escape
- Bloating mechanisms
- Detailed equation of state for the core
- Cooling of the core

Additional material

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- 6 Formation tracks in the mass–semi-major axis plane**
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Formation tracks



Locally isothermal
 Type I, unsaturated
 Type I, saturated
 Type II
 ($d \ln M / d \ln a = -\pi$)

Mordasini et al.

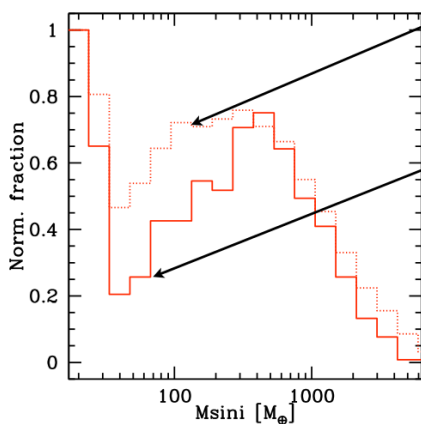
$1 M_{\odot}$, $\alpha = 7 \times 10^{-3}$, non-isothermal Type I (no reduction), 1 seed per disc

Additional material

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Accretion rate effect

Dependence on gas accretion rate in runaway



Planetary gas accretion rate limited to slow viscous disk accretion rate.

Shallow minimum.

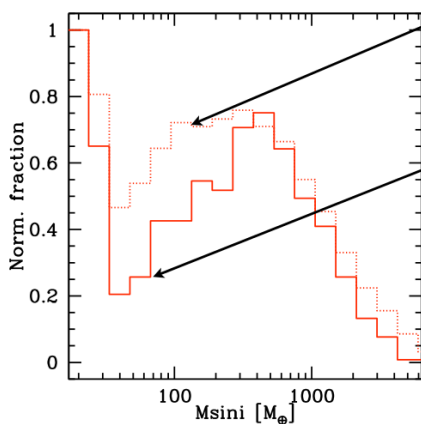
Planetary gas accretion rate **not** limited to disk accretion rate for gas already in the planet's hill sphere (fast accretion).

Deep minimum.

Mordasini et al 2011

Accretion rate effect

Dependence on gas accretion rate in runaway



Planetary gas accretion rate limited to slow viscous disk accretion rate.

Shallow minimum.

Planetary gas accretion rate **not** limited to disk accretion rate for gas already in the planet's hill sphere (fast accretion).

Deep minimum.

Detections important for *directly* constraining formation theory.

Mordasini et al 2011