UE7b Evolution of Galaxies 2012/13 2^{nd} Homework: solutions

2. Quickie: Solar abundances

The space probe SOHO was designed to monitor the activity of the Sun and follow coronal mass ejections. But it also discovered that there are many "kamikaze" comets that plunge into the Sun. Is it possible that the metallicity of the Sun had been influenced by accretion of matter from bodies of the Solar System during all its previous life? Perhaps there existed bodies that crashed into the Sun and therefore were not able to become planets ...
Let us make a rough estimate of these effects: let us assume that all the mass in all the planets of the Solar System is in the form of `metals'. Compare that amount of material with the mass of metals which is present in the Sun. If all the planets would be merged with the Sun, and the material thoroughly mixed, by how much would the solar metallicity change? Would that be detectable?
Perhaps this mixing is not complete, and the planetary material is mixed only to the photospheric layers (for a rough estimate, let us assume that a thickness of the photosphere of 400 km, and that the density is constant within the Sun)? If one took a proper model for the photosphere, how does this compare (qualitatively) to our rough estimate?

Solution:

The metallicity of the Sun is - per mass - 0.02 of the entire gas. Thus, the solar material in the form of metals has a mass of

$$M_{metals} = 2\ 10^{30} *\ 0.02 = 4\ 10^{28}\ kg$$

The mass of the planets Jupiter (2 E27 kg), Saturn (6 E26), Uranus (9 E25), and Neptune (1 E26) gives a total mass of 3 E27 kg. All the other bodies contribute rather little. This means that if all the planets' matter is mixed with the entire Sun, this would raise the Sun's metallicity by about 7.5 percent of its present value – very difficult to detect.

If only the solar photosphere were polluted, this increases by a factor from the ratio of the volumes involved:

$$\frac{4\pi \frac{R^3}{3}}{4\pi\ R^2\ H} = \frac{R}{3\ H} = \frac{7\ 10^5\ km}{400\ km} \approx 2000$$

which would make it well detectable!

If we take a proper model, the mass density in the photosphere would be less than in the constant density approximation, and thus the factor would be even greater!

3. The IMF rules the yield

Interpretation of the metallicity distribution functions of disk G dwarfs, bulge K giants, and halo globular clusters in terms of closed box models leads to the determination of the metal yields of 1.8, 0.4, and 0.025 of the solar metallicity for the Bulge, Disk, and Halo of our Galaxy. One explanation could be that the IMF in these three stellar populations differs from each other. Let us suppose that the IMF is a power-law function $\frac{dn}{dm} = \varphi(m) \propto m^{-x}$ between the mass limits 0.1 and 100 M_{sun} (Salpeter's IMF has an exponent x = -2.35 *[Here the minus sign was wrong ☹ sorry! … but what would a law $m^{+2.35}$ mean? Are massive stars more numerous than low-mass stars?]*; please note the normalization ∫ m φ dm = 1 to unity stellar mass). Let us also assume that all stars above 10 M_{sun} produce 20 percent of their initial mass in the form of metals (p(m) = 0.2), and that all stars more massive than 5 M_{sun} leave a remnant of 1.4 M_{sun} . *Stars below 5 M_{sun} shall not eject any gas [unfortunately, this sentence got lost in the editing of the instructions ☹ – sorry!]*
Compute the yield of such a stellar population as a function of the IMF exponent x. What is the yield for a Salpeter IMF, and what exponents are needed to match the derived yields of Bulge, Disk, and Halo? Furthermore, if you stick to the Salpeter exponent, which values of the upper and lower limit for the stellar masses could also explain the three values? Your opinion?

For your own checking: for Salpeter IMF the locked-up mass fraction is $\alpha \approx 0.8$.

Solution:

First we must compute the normalization A of the IMF φ(m) = A m^{-x}

$$1 = \int_{0.1}^{100} m\, \varphi(m) dm = A \int_{0.1}^{100} m^{-x+1} dm$$

which gives

$$\frac{1}{A} = \frac{100^{-x+2} - 0.1^{-x+2}}{-x+2}$$

For the Salpeter IMF one gets 1/A = 5.826 M_{sun}. The IMF itself is given by

$$\varphi(m) = A \frac{(-x+2) \quad m^{-x}}{100^{-x+2} - 0.1^{-x+2}}$$

Next we compute the locked-up mass fraction. There are two contributions for the remnants: stars below 5 M_{sun} count entirely as remnants, and stars above 5 M_{sun} leave remnants of 1.4 M_{sun}

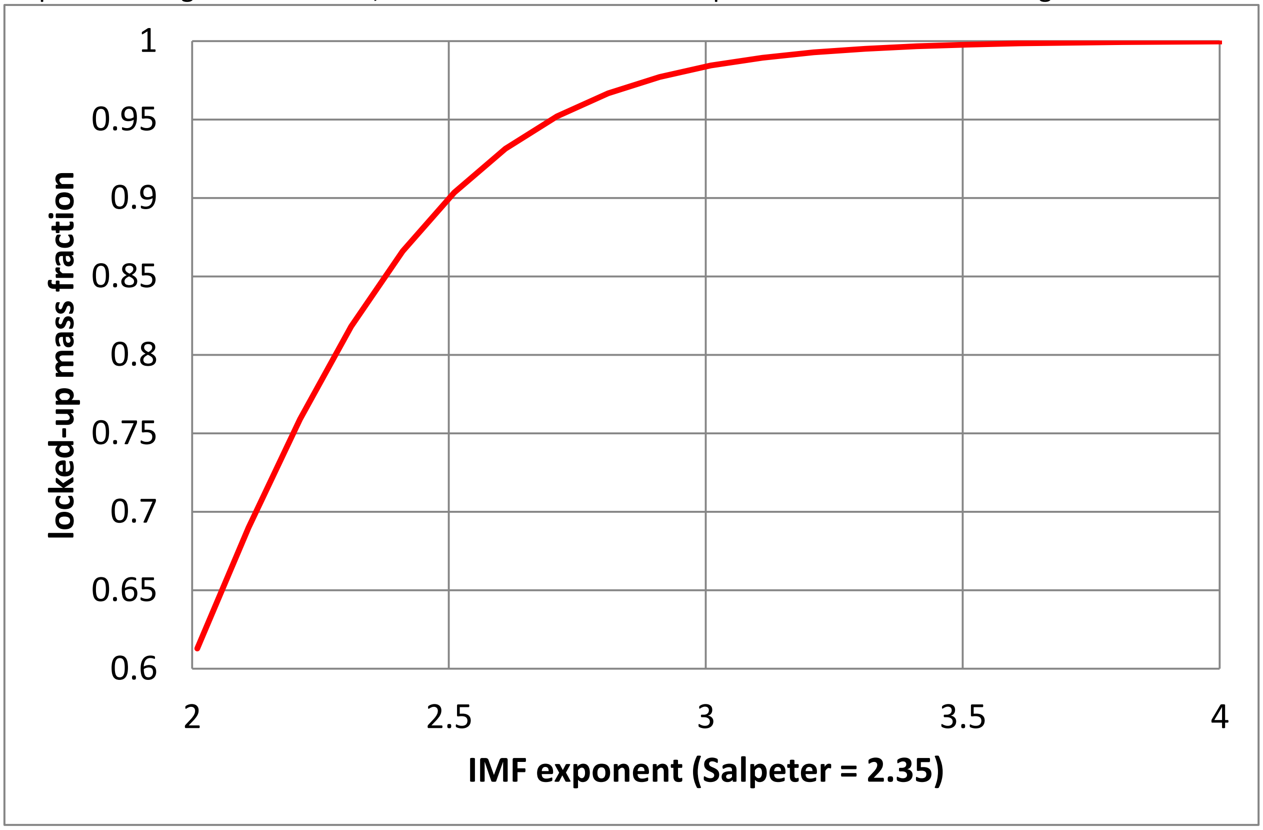
$$\alpha = \int_{0.1}^{5} m\varphi(m) dm + 1.4 \int_{5}^{100} \varphi(m) dm$$

Thus

$$\alpha = A \left[\int_{0.1}^{5} m^{-x+1} dm + 1.4 \int_{5}^{100} m^{-x} dm \right]$$

$$\alpha = A\left[\frac{5^{-x+2} - 0.1^{-x+2}}{-x+2} + 1.4\frac{100^{-x+1} - 5^{-x+1}}{-x+1}\right]$$

Salpeter's IMF gives $\alpha = 0.838$, and the results for other exponents are shown in the figure below:



If we had assumed that low-mass stars do not leave a remnant and thus return all their mass to the interstellar gas – which would be physically impossible, since these stars have longer lifetimes than massive stars and thus would be expected never to die – we would obtain:

$$\alpha = 1.4\int_{5}^{100} \varphi(m)dm$$

which for a Salpeter IMF gives a very small value for α of 0.02, and an enormously large yield!

Finally the yield is defined by

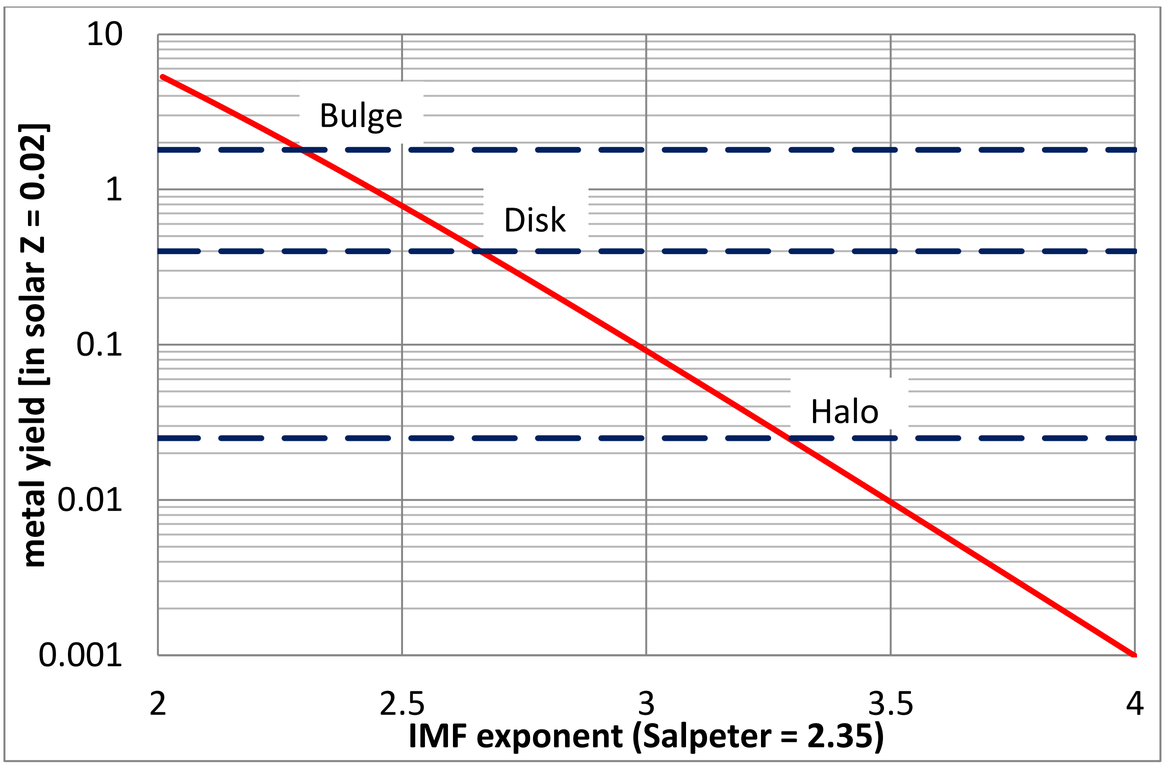
$$\alpha y = \int_{0.1}^{100} m\,p(m)\varphi(m)dm = p\int_{10}^{100} m\,\varphi(m)dm = p\,A\int_{10}^{100} m^{-x+1}dm$$

Making use of our prescription p(m) = 0.2 = p for all masses above 10 M_{sun} . This gives

$$\alpha y = p\,\frac{100^{-x+2} - 10^{-x+2}}{-x+2}$$

Results:

- The yield for a Salpeter IMF is $y = 0.0289 = 1.45\ Z_{sun}$, if we use $Z_{sun} = 0.02$.
- To match the three yield values we need x = 2.29, 2.66, and 3.29 for Bulge, Disk, and Halo. These values are within the range one could accept for observed IMFs.
- If we use Salpeter slope, but alter the upper mass limit, we would need: 282, 16, and 10.246 M_{sun} which seems rather problematic, as we would need super-massive metal-rich stars for the Bulge, we would not have any O stars (hence no HII regions) in the Disk, and a contrived mass cut for the Halo.
- If we played with the lower mass limit, we would get 0.159, 0.0043, and 0.000002 M_{sun} which is also not without problems, since it would require e.g. for the Halo a very strong production of brown dwarfs and also planets ...
- … so the easiest parameter seems to be the slope of the IMF
- The overall behaviour of the yield is shown below:



4. Quickie: Metallicity of the Sun

The metallicity (abundances of oxygen and iron) of the Sun is somewhat higher than in the nearby gas (e.g. H II regions). Someone proposed the idea that it could be so because the Sun shows the products of its own fusion processes. From your knowledge of stellar structure and evolution you can give at least two reasons why this idea cannot be true!
Also, discuss the consequences for other objects, if this idea were true, and suggest one or more observational tests with which we could confirm or refute this idea!

Solution:

- Sun is still on the main sequence, so the outer convection zone has not yet had a chance to descend into the inner region and to bring up ("dredge up") any fusion products
- the Sun on the main sequence burns only hydrogen into helium, so it has not produced any higher element yet
- the Sun is not a massive star. It can burn only hydrogen, and then helium, so it makes only helium and carbon, but no oxygen or iron
- if such an effect was true in the Sun, it should also apply to all other solar-type stars. So all other G2V stars should be overabundant. This would be seen in the spectra as stronger lines from iron and other heavy elements. Spectroscopy or photometry would reveal this.